Name: _____ Date: _____

Learning Goal 2.1	Finite limits and continuity.

A **limit** can be used to describe how a function behaves as the independent variable moves towards a certain value. But first,

The Secant Line connects 2 points on an existing graph.

Fixed Q

$m_{QP} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{4-1}{-2-1}$

$= \frac{3}{-3}$

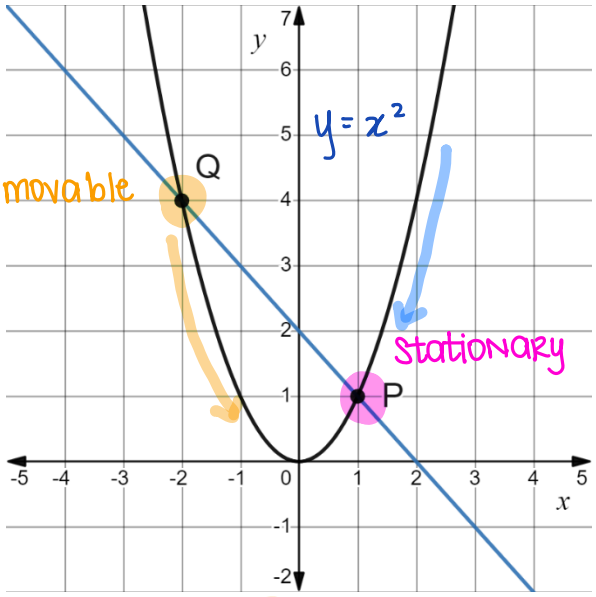
$= -1$

Movable Q (x, x^2)

$m_{QP} = \frac{x^2 - 1}{x - 1}$

↑ cannot evaluate at $x = 1$

$\{x \mid x \neq 1, x \in \mathbb{R}\}$



If we approach P from the right side:

x	m_{PQ}
2	3
1.5	2.5
1.1	2.1
1.01	2.01
1.001	2.001

If we approach P from the left side:

x	m_{PQ}
0	1
0.5	1.5
0.9	1.9
0.99	1.99
0.999	1.999

value OF the SLOPE not the FUNCTION !!

The Tangent Line is a line that just touches the GRAPH of the FUNCTION at a POINT (a, f(a)) without going THROUGH the GRAPH

$\lim_{x \to a} m_{QP}$ P(a, f(a)) (a = 1 ABOVE)

Q(x, f(x))

Assignment # 1 – 9 Quiz Next Day!

Numerically

$\lim_{x \to 1} m_{PQ} = 2$

Algebraically

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x \to 1} x + 1 = 2$$

So at (1,1) the equation of the tangent line

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x-1)$

$y - 1 = 2x - 2$

$y = 2x - 1$

Example A rock breaks loose from the top of a tall cliff. What is its average speed during the first 2 seconds of fall?

↑ the slope of the secant

(Note: Experiments show that a dense solid object dropped from rest to fall freely near the surface of the earth will fall $y = 4.9t^2$ metres in the first t seconds.)

$t = 0$, $y = 0$, ① $(0,0)$

$t = 2$, $y = 4.9(2)^2 = 19.6$, ② $(2, 19.6)$

$$m = \frac{19.6 - 0}{2 - 0} = \frac{19.6}{2} = 9.8$$

Find the speed of the rock in at the instant $t = 2$.

↑ slope of the tangent

Numerically

t	m_{PQ}
1.90	19.11
1.99	19.551
2	–
2.01	19.649
2.10	20.09

≈ 19.6

Happy dance!

Can't plug in $t = 2$

$m = \frac{4.9t^2 - 19.6}{t - 2}$

Algebraically

$$\lim_{t \to 2} m = \lim_{t \to 2} \frac{4.9t^2 - 19.6}{t - 2} = \lim_{t \to 2} \frac{4.9(t^2 - 4)}{t - 2} = \lim_{t \to 2} \frac{4.9(t+2)(t-2)}{t-2} = \lim_{t \to 2} 4.9(t+2)$$

$= 4.9(2+2)$

$= 19.6$

Algebra is preferable though we will use graphs a lot too!