Name: ________________________ Date: __________________

Learning Goal 2.1	Finite limits and continuity.

More Questions – Solutions

1. Suppose that the amount of air in a balloon after t hours is given by
$$V(t) = t^3 - 6t^2 + 35.$$
Estimate the instantaneous rate of change of the volume after 5 hours numerically. Confirm algebraically.

$$\begin{aligned} V(5) &= (5)^3 - 6(5)^2 + 35 \\ &= 125 - 150 + 35 \\ &= 10 \end{aligned}$$

$$\begin{aligned} m &= \frac{(t^3 - 6t^2 + 35) - 10}{t - 5} \\ &= \frac{t^3 - 6t^2 + 25}{t - 5} \end{aligned}$$

Numerically

x	m
4.8	13.24
4.9	14.11
4.99	14.9101
5	–
5.01	15.0901
5.1	15.91
5.2	16.84

Algebraically

$$\begin{aligned} \lim_{t\to5} m &= \lim_{t\to5} \frac{t^3 - 6t^2 + 25}{t - 5} \\ &= \lim_{t\to5} \frac{(t-5)(t^2 - t - 5)}{t - 5} \\ &= \lim_{t\to5} t^2 - t - 5 \\ &= (5)^2 - (5) - 5 \\ &= 15 \end{aligned}$$

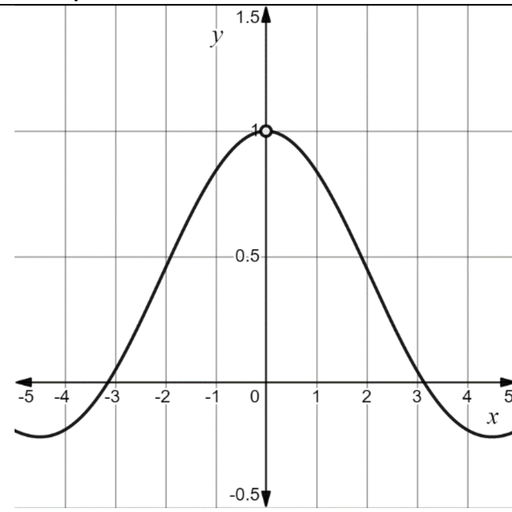
2. Consider numerically, then graphically (using technology) what happens to the y – value as the x – value gets close to zero of

$$y = \frac{\sin x}{x}$$

Numerically

x	y
-0.1	0.99833
-0.01	0.99998
-0.001	0.99999
0	$-$
0.001	0.99999
0.01	0.99998
0.1	0.99833

Graphically



$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

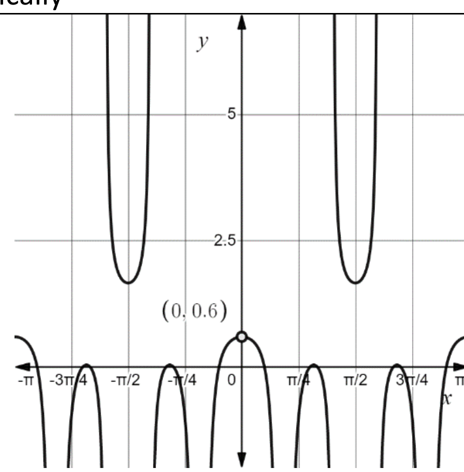
3. Consider numerically, then graphically (using technology) what happens to the y – value as the x – value gets close to zero of

$$y = \frac{\tan(3x)}{\tan(5x)}$$

Numerically

x	y
-0.1	0.56624
-0.01	0.59968
-0.001	0.59999
0	$-$
0.001	0.59999
0.01	0.59968
0.1	0.56624

Graphically



$$\lim_{x \to 0} \frac{\sin x}{x} = 0.6$$