Name: ___________________________ Date: ____________________

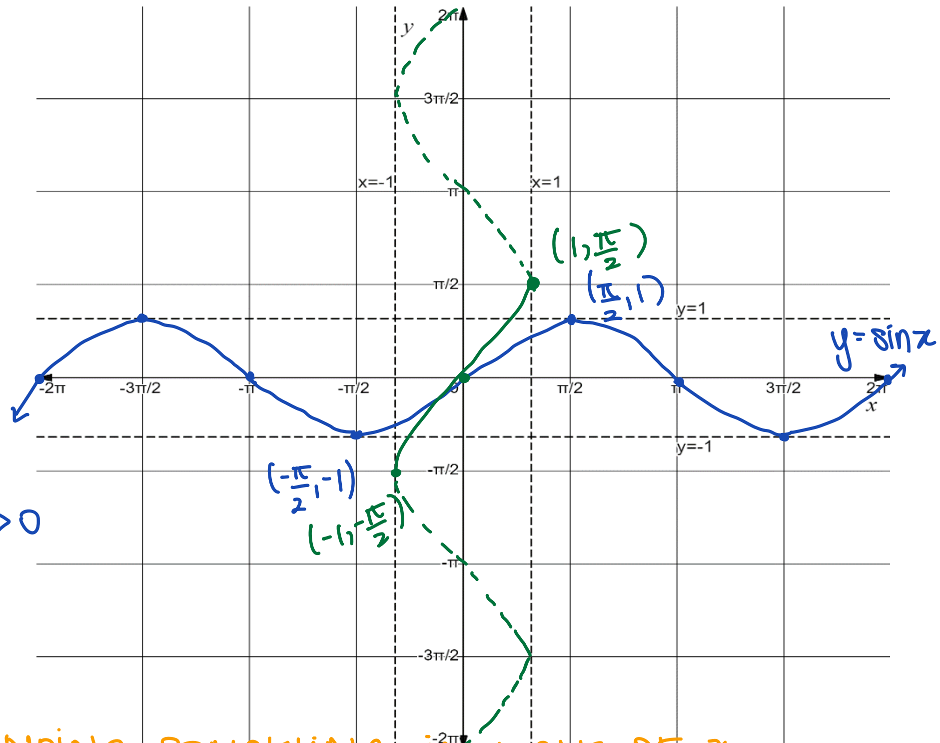
Learning Goal 3.5	Using the last derivative rules (for now).

Consider $y = \sin x$ and the

inverse function $x = \sin y$

$y = \sin^{-1} x$

$= \arcsin x$



$-\frac{\pi}{2} < y < \frac{\pi}{2}$

$-1 < x < 1$

WHY IS $\cos y > 0$

And now the derivative

$x = \sin y$

$1 = \cos y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\cos y}$

$$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}}$$

FINDING SOMETHING IN TERMS OF x FOR $\cos y$

$\sin^2 y + \cos^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \pm\sqrt{1-\sin^2 y}$

$= \pm\sqrt{1-x^2}$

Again, but with $y = \tan^{-1} x$

$x = \tan y$

$1 = \sec^2 y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+x^2}$

$\frac{\sin^2 y + \cos^2 y = 1}{\cos^2 y}$

$\tan^2 y + 1 = \sec^2 y$

$\sec^2 y = 1 + \tan^2 y$

$= 1 + x^2$

Example Differentiate.

a. $y = \dfrac{1}{\sin^{-1} x}$

$\dfrac{dy}{dx} = (\sin^{-1} x)^{-1}$

$= (\arcsin x)^{-1}$

$= -(\sin^{-1} x)^{-2} \times \dfrac{1}{\sqrt{1-x^2}}$

$= -\dfrac{1}{(\sin^{-1} x)^2 \sqrt{1-x^2}}$

b. $f(x) = \underbrace{x \arctan \sqrt{x}}_{\text{CHAIN}}$

PRODUCT

$f'(x) = \arctan\sqrt{x} + x\left(\dfrac{1}{1+(\sqrt{x})^2} \times \dfrac{1}{2}x^{-1/2}\right)$

$= \arctan\sqrt{x} + \dfrac{\sqrt{x}}{2(1+x)}$

Derivatives of Inverse Trigonometric Functions

$\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$ $\quad \dfrac{d}{dx}(\cos^{-1} x) = -\dfrac{1}{\sqrt{1-x^2}}$ $\quad \dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$

$\dfrac{d}{dx}(\csc^{-1} x) = -\dfrac{1}{|x|\sqrt{x^2-1}}$ $\quad \dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{|x|\sqrt{x^2-1}}$ $\quad \dfrac{d}{dx}(\cot^{-1} x) = -\dfrac{1}{1+x^2}$