Name: _____ Date: _____

Learning Goal 3.5	Using the last derivative rules (for now).

More Questions – Solutions

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\cos^{-1}x)=-\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1}x)=\frac{1}{1+x^2}$$

$$\frac{d}{dx}(\csc^{-1}x)=-\frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}(\sec^{-1}x)=\frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}(\cot^{-1}x)=-\frac{1}{1+x^2}$$

Example Differentiate.

a. $f(x)=\log_2(\sin^{-1}(x^2-3x))$

$$f'(x)=\frac{1}{\ln 2\,(\sin^{-1}(x^2-3x))}\times\frac{1}{\sqrt{1-(x^2-3x)^2}}\times(2x-3)$$
$$=\frac{2x-3}{\ln 2\,(\sin^{-1}(x^2-3x))\sqrt{1-(x^2-3x)^2}}$$

b. $\cos^{-1}(xy)=x^2$

$$-\frac{1}{\sqrt{1-(xy)^2}}\times\left(y+x\frac{dy}{dx}\right)=2x$$
$$y+x\frac{dy}{dx}=-2x\sqrt{1-(xy)^2}$$
$$x\frac{dy}{dx}=-2x\sqrt{1-(xy)^2}-y$$
$$\frac{dy}{dx}=-2\sqrt{1-(xy)^2}-\frac{y}{x}$$

c. $f(x)=\csc^{-1}(5x^2+1)$

$$f'(x)=-\frac{1}{(5x^2+1)\sqrt{(5x^2+1)^2-1}}\times 10x$$
$$=-\frac{10x}{(5x^2+1)\sqrt{(5x^2+1)^2-1}}$$

d. $g(x)=\sqrt{e^{\cos^{-1}x}}$

$$g'(x)=\frac{1}{2}\left(e^{\cos^{-1}x}\right)^{-1/2}\times e^{\cos^{-1}x}\times-\frac{1}{\sqrt{1-x^2}}$$
$$=\frac{1}{2}\left(e^{\cos^{-1}x}\right)^{1/2}\times-\frac{1}{\sqrt{1-x^2}}$$
$$=-\frac{\sqrt{e^{\cos^{-1}x}}}{2\sqrt{1-x^2}}$$

e. $\tan^{-1}(x-y) = xy$

$$\frac{1}{1+(x-y)^2} \times \left(1 - \frac{dy}{dx}\right) = y + x\frac{dy}{dx}$$

$$\frac{1}{1+(x-y)^2} - \frac{1}{1+(x-y)^2}\frac{dy}{dx} = y + x\frac{dy}{dx}$$

$$\frac{1}{1+(x-y)^2} - y = x\frac{dy}{dx} + \frac{1}{1+(x-y)^2}\frac{dy}{dx}$$

$$\frac{1}{1+(x-y)^2} - y = \left(x + \frac{1}{1+(x-y)^2}\right)\frac{dy}{dx}$$

$$\frac{1}{1+(x-y)^2} - y = \left(x + \frac{1}{1+(x-y)^2}\right)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\frac{1}{1+(x-y)^2} - y}{x + \frac{1}{1+(x-y)^2}}$$

$$\frac{dy}{dx} = \frac{\frac{1 - y(1+(x-y)^2)}{1+(x-y)^2}}{\frac{x(1+(x-y)^2)+1}{1+(x-y)^2}}$$

$$\frac{dy}{dx} = \frac{1 - y(1+(x-y)^2)}{x(1+(x-y)^2)+1}$$

f. $h(x) = \cos^{-1}(\log_2 x)$

$$h'(x) = -\frac{1}{\sqrt{1-(\log_2 x)^2}} \times \frac{1}{x \ln 2}$$

$$= -\frac{1}{x \ln 2 \sqrt{1-(\log_2 x)^2}}$$