

Name: _____

Date: _____

Learning Goal 3.3

Using more derivative rules.

$$\ln x = \log_e x$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\ln(g(x))) = \frac{1}{g(x)} \times g'(x)$$

↑
CHAIN RULE

$$\frac{d}{dx}(\log_b(x)) = \frac{1}{\ln b \times x}$$

$$\frac{d}{dx}(\log_b(g(x))) = \frac{1}{\ln b \times g(x)} \times g'(x)$$

**** Recall the domain for logarithms **** $x > 0$

(untransformed)

Example Differentiate.

a. $y = \cos(\ln x)$

$$\frac{dy}{dx} = -\sin(\ln x) \times \frac{1}{x}$$

$$= -\frac{\sin(\ln x)}{x}$$

$$x > 0$$

b. $y = (\ln(1 + e^x))^2$

$$\frac{dy}{dx} = 2(\ln(1 + e^x))' \times \frac{d}{dx}(\ln(1 + e^x))$$

$$= 2(\ln(1 + e^x)) \times \frac{1}{1 + e^x} \times \frac{d}{dx}(1 + e^x)$$

$$= 2(\ln(1 + e^x)) \times \frac{1}{1 + e^x} \times e^x$$

Change of Base for Logarithms

$$\log_b x = \frac{\log_a x}{\log_a b}$$

* a is arbitrary

* mostly used for evaluating

$$c. \quad y = \ln \sqrt{\frac{3x+2}{3x-2}}$$

$$f(x) = \frac{3x+2}{3x-2}$$

$$\begin{aligned} f'(x) &= \frac{(3x-2)(3x+2)' - (3x+2)(3x-2)'}{(3x-2)^2} \\ &= \frac{3(3x-2) - 3(3x+2)}{(3x-2)^2} \\ &= \frac{-12}{(3x-2)^2} \end{aligned}$$

$$d. \quad y = \log_2(1-3x)$$

$$\frac{dy}{dx} = \frac{1}{\ln 2(1-3x)} \times -3$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\sqrt{\frac{3x+2}{3x-2}}} \times \frac{1}{2\sqrt{\frac{3x+2}{3x-2}}} \times \frac{-12}{(3x-2)^2} \\ &= \frac{-12}{2\left(\frac{3x+2}{3x-2}\right) \times (3x-2)^2} \\ &= \frac{-6}{(3x+2)(3x-2)} \end{aligned}$$