

Name: _____

Date: _____

Learning Goal 3.3

Using more derivative rules.

More Questions – Solutions

1. Differentiate.

a. $y = \ln(x^4 \sin^2 x)$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{x^4 \sin^2 x} \times (x^4 \sin^2 x)' \\ &= \frac{1}{x^4 \sin^2 x} \times (4x^3 \sin^2 x + 2x^4 \sin x \cos x) \\ &= \frac{1}{x^4 \sin^2 x} \times 2x^3 \sin x (2 \sin x + x \cos x) \\ &= \frac{1}{x \sin x} \times 2(2 \sin x + x \cos x) \\ &= \frac{2(2 \sin x + x \cos x)}{x \sin x} \end{aligned}$$

b. $y = (\ln(1 + e^x))^2$

$$\begin{aligned} \frac{dy}{dx} &= 2 \ln(1 + e^x) \times (\ln(1 + e^x))' \\ &= 2 \ln(1 + e^x) \times \frac{1}{1 + e^x} \times (1 + e^x)' \\ &= \frac{2e^x \ln(1 + e^x)}{1 + e^x} \end{aligned}$$

c. $y = \ln(x + \sqrt{1 + x^2})$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{x + \sqrt{1 + x^2}} \times (x + \sqrt{1 + x^2})' \\ &= \frac{1}{x + \sqrt{1 + x^2}} \times \left(1 + \frac{1}{2\sqrt{1 + x^2}}\right) \times (1 + x^2)' \\ &= \frac{1}{x + \sqrt{1 + x^2}} \times \left(1 + \frac{1}{2\sqrt{1 + x^2}}\right) \times 2x \\ &= \frac{2x}{x + \sqrt{1 + x^2}} \left(1 + \frac{1}{2\sqrt{1 + x^2}}\right) \end{aligned}$$

d. $y = \frac{\ln x}{x^2}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{x^2(\ln x)' - \ln x (x^2)'}{(x^2)^2} \\ &= \frac{x^2 \left(\frac{1}{x}\right) - \ln x (2x)}{x^4} \\ &= \frac{x - 2x \ln x}{x^4} \\ &= \frac{1 - 2 \ln x}{x^3} \end{aligned}$$

$$\text{e. } y = \log\left(\frac{x}{x-1}\right)$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\ln(10)} \times \frac{1}{\left(\frac{x}{x-1}\right)} \times \left(\frac{x}{x-1}\right)', \\ &= \frac{1}{\ln(10)} \times \frac{1}{\left(\frac{x}{x-1}\right)} \times \frac{(x-1)(x)' - x(x-1)'}{(x-1)^2} \\ &= \frac{1}{\ln(10)} \times \frac{x-1}{x} \times \frac{(x-1)(1) - x(1)}{(x-1)^2} \\ &= \frac{1}{\ln(10)} \times \frac{x-1}{x} \times \frac{-1}{(x-1)^2} \\ &= \frac{1}{\ln(10)} \times \frac{1}{x} \times \frac{-1}{x-1} \\ &= \frac{-1}{x(x-1)\ln(10)} \end{aligned}$$

$$\text{f. } y = 2x \log \sqrt{x}$$

$$\begin{aligned} \frac{dy}{dx} &= 2x(\log \sqrt{x})' + \log \sqrt{x} (2x)' \\ &= 2x \times \frac{1}{\ln 10} \times \frac{1}{\sqrt{x}} \times (\sqrt{x})' + 2 \log \sqrt{x} \\ &= 2x \times \frac{1}{\ln 10} \times \frac{1}{\sqrt{x}} \times \frac{1}{2\sqrt{x}} + 2 \log \sqrt{x} \\ &= x \times \frac{1}{\ln 10} \times \frac{1}{\sqrt{x}} \times \frac{1}{\sqrt{x}} + 2 \log \sqrt{x} \\ &= x \times \frac{1}{\ln 10} \times \frac{1}{x} + 2 \log \sqrt{x} \\ &= \frac{1}{\ln 10} + 2 \log \sqrt{x} \end{aligned}$$