

Name: _____

Date: _____

Learning Goal 4.1

Examining angles in standard position in both radians and degrees. Exploring the unit circle, reference and coterminal angles and special angles.

1. Convert to radians, leave your answer as an exact value.

$$\begin{aligned} \text{a. } 30^\circ &\times \frac{\pi}{180} \\ &= 1 \times \frac{\pi}{6} \\ &= \frac{\pi}{6} \end{aligned}$$

$$\begin{aligned} \text{b. } 315^\circ &\times \frac{\pi}{180} \\ &= 7 \times \frac{\pi}{4} \\ &= \frac{7\pi}{4} \end{aligned}$$

2. Convert to radians, round your answer to the nearest hundredth.

$$\begin{aligned} \text{a. } 123^\circ &\times \frac{\pi}{180} \\ &= \frac{123\pi}{180} \\ &\approx 2.15 \end{aligned}$$

$$\begin{aligned} \text{b. } 257^\circ &\times \frac{\pi}{180} \\ &= \frac{257\pi}{180} \\ &\approx 4.49 \end{aligned}$$

3. Convert to degrees, round your answer to the nearest degree.

$$\begin{aligned} \text{a. } \frac{3\pi}{2} &\times \frac{180}{\pi} \\ &= \frac{3}{2} \times \frac{180}{1} \\ &= \frac{3}{1} \times \frac{90}{1} \\ &= 270^\circ \end{aligned}$$

$$\begin{aligned} \text{b. } \frac{4\pi}{5} &\times \frac{180}{\pi} \\ &= \frac{4}{5} \times \frac{180}{1} \\ &= \frac{4}{1} \times \frac{36}{1} \\ &= 144^\circ \end{aligned}$$

$$\begin{aligned} \text{c. } 1.5 &\times \frac{180}{\pi} \\ &= \frac{270}{\pi} \\ &\approx 86^\circ \end{aligned}$$

4. Draw each angle in standard position. Find the reference angle. Determine one positive and one negative co-terminal angle.

$$\text{a. } -115^\circ$$

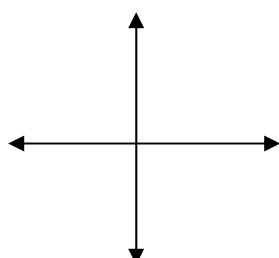
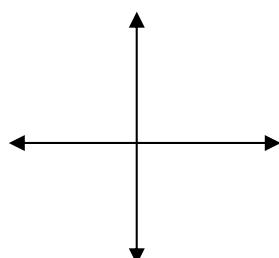
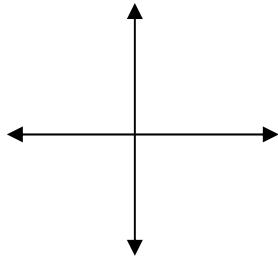
$$\begin{aligned} \theta_R &= 180^\circ - 115^\circ \\ &= 65^\circ \\ \theta_1 &= -115^\circ + 360^\circ \\ &= 245^\circ \\ \theta_2 &= -115^\circ - 360^\circ \\ &= -475^\circ \end{aligned}$$

$$\text{b. } 2$$

$$\begin{aligned} \theta_R &= \pi - 2 \\ &\approx 1.14 \\ \theta_1 &= 2 + 2\pi \\ &\approx 8.28 \\ \theta_2 &= 2 - 2\pi \\ &\approx -4.28 \end{aligned}$$

$$\text{c. } \frac{\pi}{3}$$

$$\begin{aligned} \theta_R &= \frac{\pi}{3} \\ \theta_1 &= \frac{\pi}{3} + 2\pi \\ &= \frac{7\pi}{3} \\ \theta_2 &= \frac{\pi}{3} - 2\pi \\ &= -\frac{5\pi}{3} \end{aligned}$$



5. Determine one positive and one negative co-terminal angle of the following angles. Illustrate each angle with a diagram. Write a general formula for coterminal angles in each case.

a. $\theta = 250^\circ$

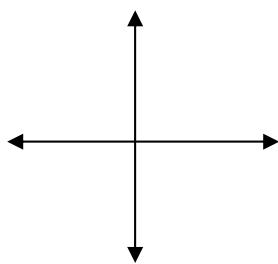
$$\begin{aligned}\theta_1 &= 250^\circ + 360^\circ \\ &= 610^\circ \\ \theta_2 &= 250^\circ - 360^\circ \\ &= -110^\circ\end{aligned}$$

b. $\theta = -315^\circ$

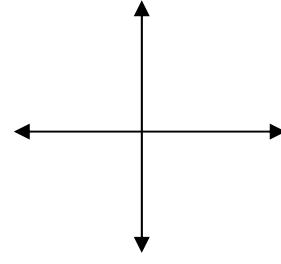
$$\begin{aligned}\theta_1 &= -315^\circ + 360^\circ \\ &= 45^\circ \\ \theta_2 &= -315^\circ - 360^\circ \\ &= -675^\circ\end{aligned}$$

c. $\theta = \frac{8\pi}{3}$

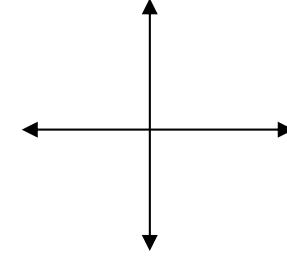
$$\begin{aligned}\theta_1 &= \frac{8\pi}{3} + 2\pi \\ &= \frac{14\pi}{3} \\ \theta_2 &= \frac{8\pi}{3} - 4\pi \\ &= -\frac{4\pi}{3}\end{aligned}$$



$$\theta = 250^\circ + 360n, n \in \mathbb{Z}$$



$$\theta = -315^\circ + 360n, n \in \mathbb{Z}$$



$$\theta = \frac{8\pi}{3} + 2\pi n, n \in \mathbb{Z}$$

6. A circle has radius 8.2 cm. Calculate the length of an arc of this circle subtended by 125° . Express the length to the nearest tenth of a centimetre.

$$\begin{aligned}a &= 2\pi r \times \frac{\theta}{360} \\ &= 2\pi(8.2) \times \frac{125}{360} \\ &\approx 17.9 \text{ cm}\end{aligned}$$

7. Determine the central angle (in radians) that is subtended by a sector of area 3 cm^2 in a circle of radius 10 cm.

$$\begin{aligned}A &= \pi r^2 \times \frac{\theta}{2\pi} = 3 \\ r^2 \times \frac{\theta}{2} &= 3 \\ (10)^2 \times \frac{\theta}{2} &= 3 \\ 100 \times \frac{\theta}{2} &= 3 \\ 50 \times \theta &= 3 \\ \theta &= \frac{3}{50} \\ \theta &\approx 0.06\end{aligned}$$