Name: ___________________________ Date: ____________________

Learning Goal 6.1	Using identities to reduce complexity in expressions and solve equations.

Identity is a statement that is always true for all values for which the identity is defined.

Example $x^2 - 9 = (x-3)(x+3)$ is an identity. Identities can be **proven** or **verified**.

a. Verify the function

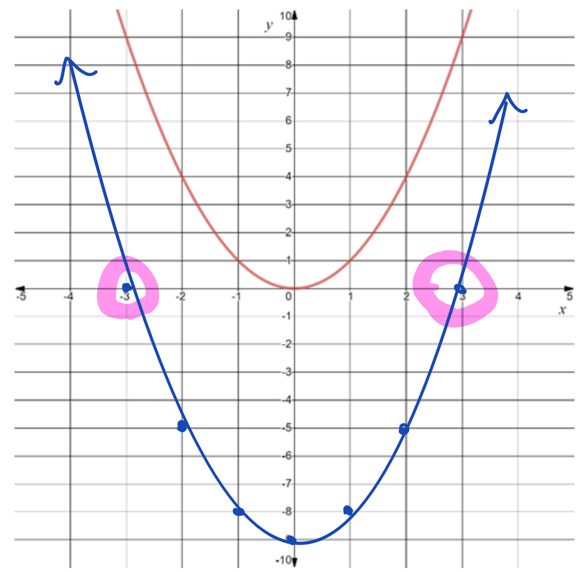
Numerically

$x = 9$

$(9)^2 - 9$
$= 81 - 9$
$= 72$

$(9-3)(9+3)$
$= (6)(12)$
$= 72$

Graphically



b. Prove the identity

$x^2 + 0x - 9$	$(x-3)(x+3)$
$= x^2 - 3x + 3x - 9$	$= x^2 + 3x - 3x - 9$
$= x(x-3) + 3(x-3)$	$= x^2 + 0x - 9$
$= (x-3)(x+3)$	$= x^2 - 9$

$-3 \times 3 = -9$
$-3 + 3 = 0$

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Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

Quotient Identities

$\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

$\csc x = \dfrac{1}{\sin x}$ $\sec x = \dfrac{1}{\cos x}$ $\cot x = \dfrac{1}{\tan x}$

Example a. Verify that $\tan^2 x + 1 = \sec^2 x$ using $x = \dfrac{\pi}{6}$

$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$ $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$

$\left(\frac{1}{\sqrt{3}}\right)^2 + 1 \overset{?}{=} \left(\frac{2}{\sqrt{3}}\right)^2$

$\frac{1}{3} + \cancel{1}\,\frac{3}{3} = \frac{4}{3}$

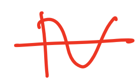
$\frac{4}{3} = \frac{4}{3}$

b. Verify that $\tan^2 x + 1 = \sec^2 x$ could be an identity by starting from the original Pythagorean identity.

$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$\tan^2 x + 1 = \sec^2 x$

denominator = 0



Example State any restrictions (non-permissible values) for the identity $\dfrac{\cot x}{\csc x \cos x}$ then simplify.

$\cos x \neq 0$ $\csc x \neq 0$

$\boxed{x \neq \frac{\pi}{2}, \frac{3\pi}{2}}$ $\frac{1}{\sin x} \neq 0$

no values $1 \neq 0$

$\cot x = \frac{\cos x}{\sin x}$

$\sin x \neq 0$

$\boxed{x \neq 0, \pi, 2\pi}$

$= \dfrac{\left(\frac{\cos x}{\sin x}\right)}{\left(\frac{1}{\sin x}\right)(\cos x)}$

$= \dfrac{\left(\frac{\cos x}{\sin x}\right)}{\left(\frac{\cos x}{\sin x}\right)} = \frac{\cancel{\cos x}}{\cancel{\sin x}} \times \frac{\cancel{\sin x}}{\cancel{\cos x}} = 1$

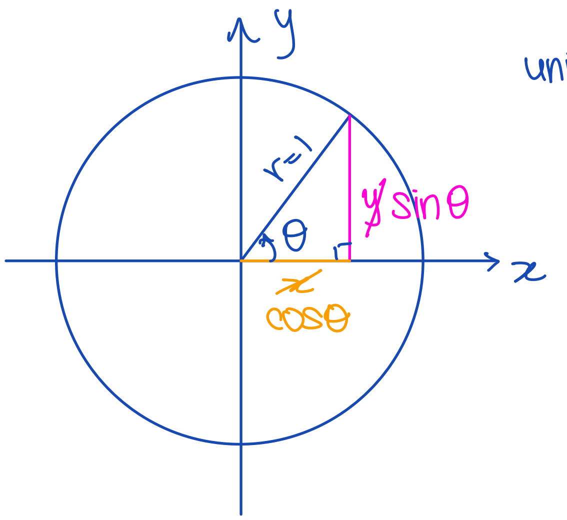
Example Prove $\mathbf{1 + \cot^2 x = \csc^2 x}$.

$1 + \cot^2 x$	$\csc^2 x$
$= 1 + \frac{\cos^2 x}{\sin^2 x}$	$= \frac{1}{\sin^2 x}$ ← decomp.
$= \frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x}$	
$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x}$	
$= \frac{1}{\sin^2 x}$	

$1 = \frac{\sin^2 x}{\sin^2 x}$

Pythagorean identity

Assignment p. 296 # 1 – 12, 14 – 17, C1, C2 Quiz Next Day!



unit circle

$$\overset{\cos^2\theta}{\cancel{x^2}} + \overset{\sin^2\theta}{\cancel{y^2}} = 1$$

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$

$$\sin\theta = y \qquad \cos\theta = x$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$