Name: ________________________ Date: __________________

Learning Goal 6.1	Using identities to reduce complexity in expressions and solve equations.

More Questions - Solutions

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

Quotient Identities

$\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

$\csc x = \dfrac{1}{\sin x}$ $\sec x = \dfrac{1}{\cos x}$ $\cot x = \dfrac{1}{\tan x}$

1. State any restrictions (non-permissible values) in radians for the following identities then simplify.

a. $\dfrac{\sec x}{\tan x}$

$\tan x \neq 0$

$\{x | x \neq 0 + n\pi, n \in \mathbb{Z}, x \in \mathbb{R}\}$

$$= \frac{1}{\cos x} \Big/ \frac{\sin x}{\cos x}$$
$$= \frac{1}{\cos x} \times \frac{\cos x}{\sin x}$$
$$= \frac{1}{\sin x}$$
$$= \csc x$$

b. $\dfrac{\sin x + \tan x}{1 + \cos x}$

$1 + \cos x \neq 0$

$\cos x \neq -1$

$\{x | x \neq \pi + 2\pi n, n \in \mathbb{Z}, x \in \mathbb{R}\}$

$$= \frac{\sin x + \sin x / \cos x}{1 + \cos x}$$
$$= \frac{\cos x \sin x + \sin x / \cos x}{1 + \cos x}$$
$$= \frac{\cos x \sin x + \sin x}{\cos x} \times \frac{1}{1 + \cos x}$$
$$= \frac{\sin x (\cos x + 1)}{\cos x} \times \frac{1}{1 + \cos x}$$
$$= \frac{\sin x}{\cos x}$$
$$= \tan x$$

c. $\dfrac{\csc x - \sin x}{\cot x}$

$\cot x \neq 0$

$\{x | x \neq \dfrac{\pi}{2} + n\pi, n \in \mathbb{Z}, x \in \mathbb{R}\}$

$$= \frac{1/\sin x - \sin x}{\cos x / \sin x}$$
$$= \frac{1 - \sin^2 x / \sin x}{\cos x / \sin x}$$
$$= \frac{1 - \sin^2 x}{\sin x} \times \frac{\sin x}{\cos x}$$
$$= \frac{1 - \sin^2 x}{\cos x}$$
$$= \frac{\cos^2 x}{\cos x}$$
$$= \cos x$$

2. Prove $\tan^2 x + 1 = \sec^2 x$.

$\tan^2 x + 1$	$\sec^2 x$
$= \dfrac{\sin^2 x}{\cos^2 x} + 1$ $= \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x}$ $= \dfrac{1}{\cos^2 x}$ $= \sec^2 x$	