

Name: \_\_\_\_\_

Date: \_\_\_\_\_

<b>Learning Goal 8.1</b>	Solving exponential and logarithmic equations with same base and with different bases, including base $e$ .
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**Example** Write each expression without brackets and with positive exponents.

a.  $\frac{24m^5p^{-3}q^4}{-4m^4p^2q^{-2}}$

$= \frac{6m^5p^{-3}q^4}{-m^4p^2q^{-2}}$

$= \frac{6mp^{-3}q^4}{-p^2q^{-2}}$

$= \frac{6mq^4}{-p^3q^{-2}}$

$= -\frac{6mq^6}{p^3}$

$\frac{m^5}{m^4} = m^{5-4} = m$

$\frac{p^{-3}}{p^2} = p^{-3-2} = p^{-5} = \frac{1}{p^5}$

b.  $\left(\frac{18x^{-2}y^3}{54x^{-6}y^{-1}}\right)^3 \div \frac{(6x^2y^{-3})^{-2}}{(x^{-4}y^2)^3}$

$= \left(\frac{x^4y^3y}{3x^2}\right)^3 \div \frac{1}{(x^{-4}y^2)^3(6x^2y^{-3})^2}$

$= \left(\frac{x^4y^4}{3}\right)^3 \times (x^{-4}y^2)^3(6x^2y^{-3})^2$

$= \frac{x^{12}y^{12}}{3^3} \times x^{-12}y^6 \times 6^2x^4y^{-6}$

$= \frac{6^2x^4y^{12}}{3^3} = \frac{4x^4y^{12}}{3}$

BEDMAS

**Example** Convert each of the following to the base indicated.

a.  $32^x$  to base 2

$= (2^5)^x$

$= 2^{5x}$

b.  $81^{x-2}$  to base 3

$= (3^4)^{x-2}$

$= 3^{4(x-2)}$

$= 3^{4x-8}$

c.  $\frac{1}{64^{2x}}$  to base 4

$= (4^{-3})^{2x}$

$= 4^{-6x}$

**Example** Simplify the following by converting each term to a common base.

a.  $\frac{8^{3x-4} \cdot 16^{4-x}}{64^{1-2x}}$

$= \frac{(2^3)^{3x-4} (2^4)^{4-x}}{(2^6)^{1-2x}}$

$= \frac{2^{9x-12} 2^{16-4x}}{2^{6-12x}}$

$= \frac{2^{5x+4}}{2^{6-12x}} = 2^{17x-2}$

b.  $(9^{2x+3} \div 27^{3x-1}) \cdot 81^{x-1}$

$= ((3^2)^{2x+3} \div (3^3)^{3x-1}) \times (3^4)^{x-1}$

$= (3^{4x+6} \div 3^{9x-3}) \times 3^{4x-4}$

$= 3^{-5x+9} \times 3^{4x-4}$

$= 3^{-x+5}$

$= 3^{5-x}$

**Definition** An exponential equation is an equation where the variable appears in the exponent.

$$y = b^x$$

(For now, we are only solving **exponential equations with the same base** using algebraic methods)

**Example** Solve and check.

a.  $2^{(4x-1)} = 8^{2x}$

$$2^{4x-1} = (2^3)^{2x}$$

$$2^{4x-1} = 2^{6x}$$

$$\begin{matrix} 4x - 1 & = & 6x \\ -4x & & -4x \end{matrix}$$

$$\frac{-1}{2} = \frac{2x}{2}$$

$$x = -\frac{1}{2}$$

b.  $27^{x-4} = \left(\frac{1}{9}\right)^{2x-8}$

$$(3^3)^{x-4} = \left(\frac{1}{3^2}\right)^{2x-8}$$

$$3^{3x-12} = \left(\frac{1}{3}\right)^{4x-16}$$

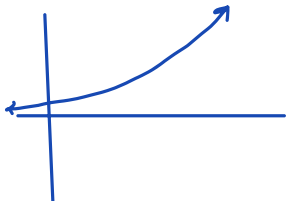
$$3^{3x-12} = 3^{-4x+16}$$

$$3x - 12 = -4x + 16$$

$$7x - 12 = 16$$

$$7x = 28$$

$$x = 4$$



- Steps**

  - make the Bases match
  - cancel Bases / equate the exponents.
  - solve.

**Example** A population of ants starts with 4000. After 4 weeks the estimated count is 128000 ants in the colony. What is the doubling period for this population?

$$I = I_0 (g)^{t/d}$$

↑ END CONDITIONS  
↑ initial conditions  
↑ GROWTH Rate  
↑ GROWTH rate time SPAN  
↑ ELAPSED time

$$128\,000 = 4000 (2)^{4/d}$$

$$32 = (2)^{4/d}$$

$$2^5 = 2^{4/d}$$

$$5 = \frac{4}{d} \Rightarrow d = \frac{4}{5}$$

$$5d = 4$$

THE DOUBLING PERIOD OF THE COLONY IS  $\frac{4}{5}$  OF a week.