Name: _____   Date: _____

Learning Goal 3.6	Linear Approximations.

Linear Approximation

- USED to APPROXIMATE THE VALUE OF A
  FUNCTION at SOME x VALUE BY USING
  tHE tANGENT AS AN APPROXIMATION
  OF tHE FUNCTION
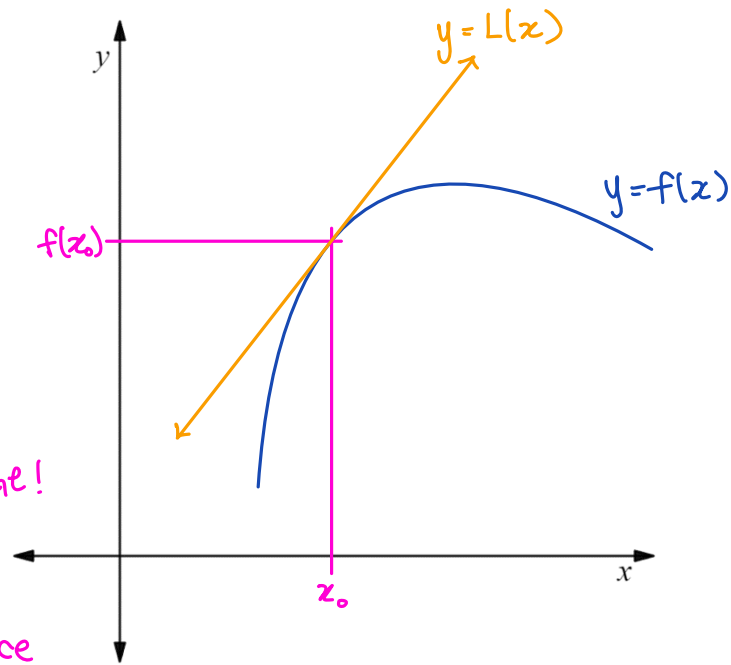   ↳ RELATIVELY ACCURATE
      ↳ eASIER to CALCULATE

   (we'Ve DONE tHIS QUITE A BIT!)

   $y - y_1 = m(x - x_1)$ SLOPE-Pt FORM
                              ↓ REARRANGE!
   $y - f(x_0) = f'(x_0)(x - x_0)$

   $y = f(x_0) + f'(x_0)(x - x_0)$
          ↗   ↗   ↗
     fUNCTION   SLOPE @   distance
     @ NICE Pt.   NICE Pt.   Dtwn NICE
                              ¿ NOt SO NICE Pt.

   ONCE we HAVE tHE LiNEAR
   APPROXIMATION, we CAN USE
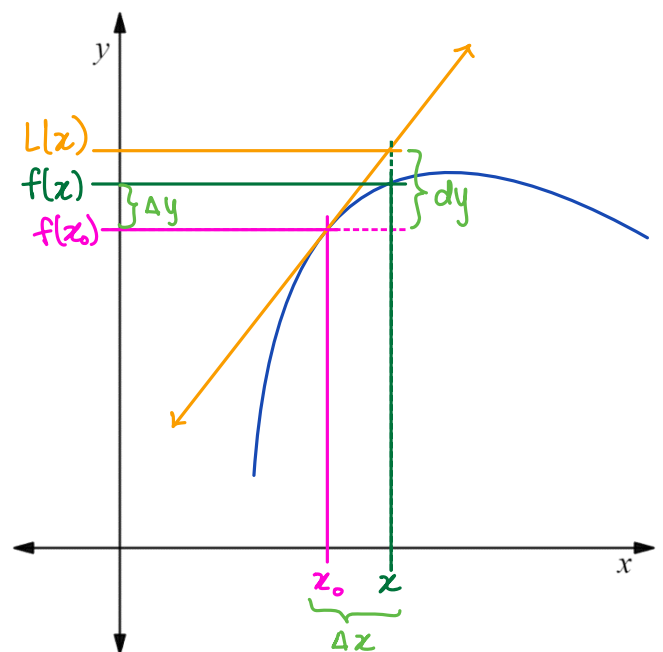   it to FIND VALUES CLOSE to x_0

ANOTHER NOtATIONAL iDEA

   $dx \times \dfrac{dy}{dx} = f'(x) \times dx$ (KIND OF...)

   $dy = f'(x) \, dx$

AND iF $\Delta y \approx f'(x) \, dx$

$\Delta y \approx dy$

$y = L(x)$

$y = f(x)$

$f(x_0)$

x_0

$L(x)$

$f(x)$ $\}\Delta y$

$f(x_0)$

$\}dy$

x_0 x

Δx

Assignment   # 1 – 7, 15 - 29   Quiz Next Day!

Example Find the linearization of $f(x) = \sqrt{x}$ at $x_0 = 4$ and use it to approximate the value of $\sqrt{3.8}$ without a calculator.

Nice   *Not nice*

$$f(x) = \sqrt{x} \qquad f(x_0) = \sqrt{4}$$
$$= 2$$

$$f(3.8) \approx f(4) + f'(4)(3.8 - 4)$$
$$\approx 2 + \frac{1}{4}(-0.2)$$

$$f'(x) = \frac{1}{2\sqrt{x}} \qquad f'(x_0) = \frac{1}{2\sqrt{4}}$$
$$= \frac{1}{4}$$

$$\approx 2 + \frac{1}{4}\left(-\frac{1}{5}\right)$$
$$\approx 2 - \frac{1}{20}$$
$$\approx 1.95 \quad \left.\right) \text{DUMB CALC ... OR NOT}$$

Example Approximate the value of $\cos 32°$. ← *Nice trig value close to 32°?*

$$f(x) = \cos x$$
$$x_0 = 30°$$

$$f(30°) = \frac{\sqrt{3}}{2}$$

$$\cos(32°) \approx \frac{\sqrt{3}}{2} - \frac{1}{2}(32 - 30)$$
$$\approx \frac{\sqrt{3}}{2} - \frac{1}{2}(2°)$$

$$f'(x) = -\sin x$$
$$f'(30°) = -\frac{1}{2}$$

$$\approx \frac{\sqrt{3}}{2} - i\left(\frac{\pi}{180°}\right) \quad \left.\right) \text{DUMB CALC}$$
$$\approx 0.85$$

Example Find the linearization of the function $f(x) = \sqrt{x + 3}$ at $x_0 = 1$. Determine the values for x for which the linear approximation is accurate to within 0.1.

We're finding the approx. values.

$$f(x) = \sqrt{x+3} \qquad f(1) = \sqrt{4} = 2$$

$$L(x) = 2 + \frac{1}{4}(x-1)$$

$$f'(x) = \frac{1}{2\sqrt{x+3}} \qquad f'(1) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$\left(\sqrt{x+3} - 0.1 < 2 + \frac{1}{4}(x-1) < \sqrt{x+3} + 0.1\right) \times 4$$

$$4\sqrt{x+3} - 0.4 < 8 + (x-1) < 4\sqrt{x+3} + 0.4$$

$$4\sqrt{x+3} - 0.4 < x + 7 \quad \checkmark$$
$$4\sqrt{x+3} < x + 7.4$$
$$16(x+3) < x^2 + 14.8x + 54.76$$
$$16x + 48 < x^2 + 14.8x + 54.76$$
$$0 < x^2 - 1.2x + 6.76$$
$$\text{QF} \quad \text{NO REAL SOLUTIONS}$$

$$x + 7 < 4\sqrt{x+3} + 0.4$$
$$x + 6.6 < 4\sqrt{x+3}$$
$$x^2 + 13.2x + 43.56 < 16(x+3)$$
$$x^2 + 13.2x + 43.56 < 16x + 48$$
$$x^2 - 2.8x - 4.94 < 0$$
$$\text{QF} \quad x = -1.1 \qquad x = 3.9$$

$$-1.1 < x < 3.9$$