

Name: _____

Date: _____

Learning Goal 3.6

Linear Approximations.

More Questions – Solutions

1. Approximate the value of
- $\sqrt{51}$
- without a calculator.

$$f(x) = \sqrt{x}$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$x_0 = 49$$

$$x = 51$$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$$\sqrt{51} \approx \sqrt{49} + \frac{1}{2\sqrt{49}}(51 - 49)$$

$$\approx 7 + \frac{1}{14}(2)$$

$$\approx 7 + \frac{1}{7}$$

$$\approx \frac{50}{7}$$

2. Approximate the value of
- $\sin 87^\circ$
- .

$$f(x) = \sin x$$

$$f'(x) = \cos x$$

$$x_0 = 90^\circ$$

$$x = 87^\circ$$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$$\sin 87^\circ \approx \sin 90^\circ + \cos 90^\circ(87 - 90)$$

$$\approx 1 + 0(-3)$$

$$\approx 1$$

$$\approx \frac{50}{7}$$

3. Given that
- $f(5) = 3$
- and
- $f'(5) = \frac{1}{2}$
- , approximate the value of
- $f(5.02)$
- .

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$$\approx 3 + \frac{1}{2}(5.02 - 5)$$

$$\approx 3 + \frac{1}{2}(0.02)$$

$$\approx 3 + \frac{1}{100}$$

$$\approx 3.01$$

4. Let
- $f(x) = \sqrt{x+4}$
- . What is
- $f(6)$
- ?

$$f(x) = \sqrt{x+4} \quad x_0 = 5$$

$$f'(x) = \frac{1}{2\sqrt{x+4}} \quad x = 6$$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$$\sqrt{10} \approx \sqrt{9} + \frac{1}{2\sqrt{9}}(6 - 5)$$

$$\approx 3 + \frac{1}{6}(1)$$

$$\approx 3 + \frac{1}{6}$$

$$\approx \frac{19}{6}$$

5. Find the linear approximation of
- $\sin x$
- at
- $x = 0$
- .

$$f(x) = \sin x \quad x_0 = 0$$

$$f'(x) = \cos x \quad x = x$$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$$\sin 0 \approx \sin 0 + \cos 0(x - 0)$$

$$\approx 0 + 1(x)$$

$$\approx x$$

$$\approx \frac{50}{7}$$

6. Find the linearization of
- $f(x) = \sqrt[3]{x}$
- then use it to approximate when
- $x = 9$
- .

$$f(x) = \sqrt[3]{x} \quad x_0 = 8$$

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}} \quad x = 9$$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$$\sqrt[3]{9} \approx \sqrt[3]{8} + \frac{1}{3\sqrt[3]{8^2}}(9 - 8)$$

$$\approx 2 + \frac{1}{12}(1)$$

$$\approx 2 + \frac{1}{12}$$

$$\approx \frac{25}{12}$$