Name: _____   Date: _____

Learning Goal 3.1	Using all basic derivative rules.

More Questions – Solutions

1. Find the derivative of the following functions.

a. $f(x) = 5x^4$

$$f'(x) = 5 \times \frac{d}{dx} x^4$$
$$= 5 \times 4x^{4-1}$$
$$= 20x^3$$

b. $f(x) = \dfrac{3}{\sqrt{x}}$

$$= 3x^{-1/2}$$
$$f'(x) = 3 \times \frac{d}{dx} x^{-1/2}$$
$$= 3 \times -\frac{1}{2} x^{-1/2 - 1}$$
$$= -\frac{3}{2} x^{-3/2}$$
$$= -\frac{3}{2x^{3/2}}$$
$$= -\frac{3\sqrt{x}}{2x^2}$$

c. $f(x) = 5x^5 + 2x^4 - 7x^2 - 9$

$$f'(x) = \frac{d}{dx} 5x^5 + \frac{d}{dx} 2x^4 - \frac{d}{dx} 7x^2$$
$$- \frac{d}{dx} 9$$
$$= 5\frac{d}{dx} x^5 + 2\frac{d}{dx} x^4 - 7\frac{d}{dx} x^2 - \frac{d}{dx} 9$$
$$= 5 \times 5x^{5-1} + 2 \times 4x^{4-1} - 7 \times 2x^{2-1}$$
$$= 25x^4 + 8x^3 - 14x$$

d. $y = x^{-100}$

$$\frac{dy}{dx} = \frac{d}{dx} x^{-100}$$
$$= -100x^{-100-1}$$
$$= -100x^{-101}$$

e. $y = x^{3/4}$

$$\frac{dy}{dx} = \frac{d}{dx} x^{3/4}$$
$$= \frac{3}{4} x^{3/4 - 1}$$
$$= \frac{3}{4} x^{-1/4}$$
$$= \frac{3}{4x^{1/4}}$$
$$= \frac{3\sqrt[4]{x^3}}{4x}$$

f. $g(x) = -4x^5 + 3x^2 - \dfrac{5}{x^2}$

$$= -4x^5 + 3x^2 - 5x^{-2}$$
$$g'(x) = -4\frac{d}{dx} x^5 + 3\frac{d}{dx} x^2$$
$$- 5\frac{d}{dx} x^{-2}$$
$$= -4 \times 5x^{5-1} + 3 \times 2x^{2-1}$$
$$- 5 \times -2x^{-2-1}$$
$$= -20x^4 + 6x + 10x^{-3}$$
$$= -20x^4 + 6x + \frac{10}{x^3}$$
$$= \frac{-20x^7 + 6x^4 + 10}{x^3}$$

2. Find the slope and equation of the tangent line at $(1, 3)$ to the graph of

$$f(x) = 2x + \frac{1}{\sqrt{x}}$$

$f(x) = 2x + x^{-1/2}$

$f'(x) = 2 - \frac{1}{2}x^{-3/2}$

$\quad = 2 - \frac{1}{2}x^{-3/2}$

$\quad = 2 - \frac{1}{2x^{3/2}}$

$f'(1) = \frac{3}{2}$

$y - 3 = \frac{3}{2}(x - 1)$

$y = \frac{3}{2}x + \frac{3}{2}$

3. The demand function for a certain product is given by

$$p(x) = \frac{\sqrt{x}}{2} - \frac{x}{40} + 2\,000$$

where p is the price measured in dollars and the quantity x is measured in units.

   a. Find the rate of change of price p per thousand products with respect to quantity x.

   $$p(x) = \frac{x^{1/2}}{2} - \frac{x}{40} + 2\,000$$

   $$p'(x) = \frac{1}{2} \times \frac{1}{2x^{1/2}} - \frac{1}{40}$$

   $$p'(x) = \frac{1}{4x^{1/2}} - \frac{1}{40}$$

   b. How fast is the price changing with respect to x when $x = 25$ and $x = 400$?

   $x = 25$ $x = 400$

   $$p'(25) = \frac{1}{4(25)^{1/2}} - \frac{1}{40}$$

   $$= \frac{1}{20} - \frac{1}{40}$$

   $$= \frac{1}{40}$$

   $$p'(400) = \frac{1}{4(400)^{1/2}} - \frac{1}{40}$$

   $$= \frac{1}{80} - \frac{1}{40}$$

   $$= -\frac{1}{80}$$

   When 25 products are requested, it will cost $0.025 more to make one more.

   When 400 products are requested, it will cost $0.0125 less to make one more.