$$(x+3)(x+2) = x^2 + 5x + b$$

Name: _____        Date: _____

| **Learning Goal 3.2** | Factoring, including the factor theorem and the remainder theorem. |
|---|---|

**Recall** Long Division by dividing 6947 by 52.

| Dividend | 6947 |
|---|---|
| Quotient | 133 |
| Divisor | 52 |
| Remainder | 31 |

*answer*

$$\begin{array}{r} 133 \\ 52\overline{)6947} \\ -52\downarrow\phantom{00} \\ \overline{\phantom{0}174\phantom{0}} \\ -156\downarrow \\ \overline{\phantom{00}187} \\ -156 \\ \overline{\phantom{000}31} \end{array}$$

Write a division statement that explicitly shows the dividend, the quotient and the remainder **in two ways**.

$$\ast \quad 6947 = 52(133) + 31 \quad \ast \qquad \ast \quad \frac{6947}{52} = 133 + \frac{31}{52} \quad \ast$$

**Example** Divide the quadratic polynomial $P(x) = x^2 + 7x + 17$ by the linear polynomial $D(x) = x + 3$. Write a division statement in the following form, stating any restrictions on the variable.

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R}{D(x)} \quad \Rightarrow \quad \frac{x^2 + 7x + 17}{x+3} = (x+4) + \frac{5}{x+3}$$

$$\begin{array}{r} x + 4 \\ x+3\overline{)x^2 + 7x + 17} \\ -(x^2 + 3x)\downarrow \\ \overline{\phantom{00}4x + 17} \\ -(4x + 12) \\ \overline{\phantom{0000}5} \end{array}$$

| $P(x)$ | POLYNOMIAL DIVIDEND |
|---|---|
| $Q(x)$ | POLYNOMIAL QUOTIENT |
| $D(x)$ | POLYNOMIAL DIVISOR |
| $R$ | REMAINDER |

$$x^2 + 7x + 17 = (x+3)(x+4) + 5$$

**Example** Divide the polynomial $P(x) = 5x^3 + 10x - 13x^2 - 9$ by $x - 2$. Write a division statement and identify any restrictions on the variable.

$$\begin{array}{r} 5x^2 - 3x + 4 \\ x-2\overline{)5x^3 - 13x^2 + 10x - 9} \\ -(5x^3 - 10x^2)\downarrow \\ \overline{\phantom{00}-3x^2 + 10x} \\ -(-3x^2 + 6x) \\ \overline{\phantom{0000}4x - 9} \\ -(4x - 8) \\ \overline{\phantom{00000}-1} \end{array}$$

$$5x^3 + 10x - 13x^2 - 9 =$$
$$(x-2)(5x^2 - 3x + 4) - 1$$

**Example** Divide $2x^3 + 3x^2 - 4x + 15$ by $x + 3$ using **synthetic division.**
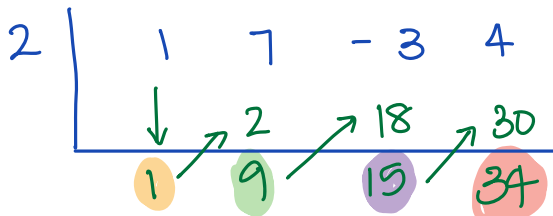
$\rightarrow (x+3)$ is a factor
$\Rightarrow x = -3$
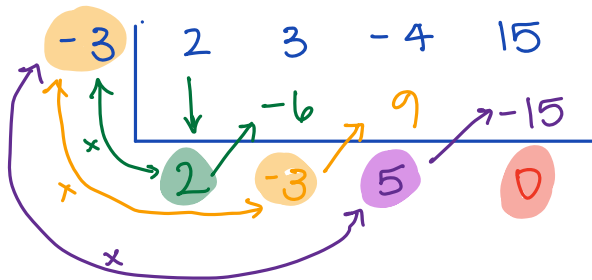


$$2x^3 + 3x^2 - 4x + 15 =$$
$$(x+3)(2x^2 - 3x + 5)$$
$$* \text{ NO REMAINDER } *$$

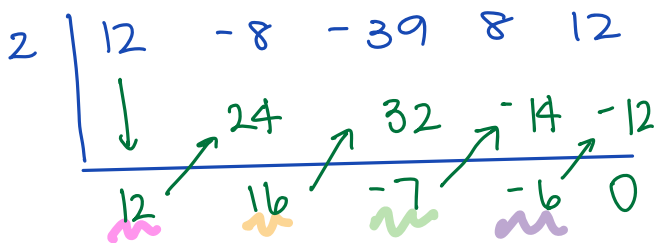**Example** Divide $x^3 + 7x^2 - 3x + 4$ by $x - 2$ using **synthetic division.**



$$x^3 + 7x^2 - 3x + 4$$
$$= (x-2)(x^2 + 9x + 15) + 34$$

*TO USE SYNTHETIC DIVISION, THE DIVISOR MUST BE A LINEAR BINOMIAL WITH A LEADING COEFFICIENT OF 1 *

**Example** Two factors of $12a^4 - 39a^2 + 8a - 8a^3 + 12$ are $a - 2$ and $2a + 1$. Find the other factors.

2 METHODS : ① USE LONG/SYNTHETIC DIVISION TWICE
② DIVIDE BY THE PRODUCT OF THE 2 FACTORS.



$(a-2)(2a+1)$
$= 2a^2 + a - 4a - 2$
$= 2a^2 - 3a - 2$

$$2a+1 \enclose{longdiv}{12a^3 + 16a^2 - 7a - 6}$$
quotient $6a^2 + 5a - 6$
$- (12a^3 + 6a^2)$
$\overline{\phantom{--}10a^2 - 7a}$
$- (10a^2 + 5a)$
$\overline{\phantom{----}-12a - 6}$
$\cdot (-12a - 6)$
$\overline{\phantom{------}0}$

$$2a^2 - 3a - 2 \enclose{longdiv}{12a^4 - 8a^3 - 39a^2 + 8a + 12}$$
quotient $6a^2 + 5a - 6$
$-(12a^4 - 18a^3 - 12a^2)$
$\overline{\phantom{--}10a^3 - 27a^2 + 8a}$
$-(10a^3 - 15a^2 - 10a)$
$\overline{\phantom{----}-12a^2 + 18a + 12}$
$-(-12a^2 + 18a + 12)$
$\overline{\phantom{------}0}$

$$(a-2)(2a+1)(6a^2 + 5a - 6) = (a-2)(2a+1)(3a-2)(2a+3)$$

$\dfrac{-4 \times 9}{-4 + 9} = \dfrac{-36}{5}$

$6a^2 - 4a + 9a - 6$

$$2a(3a-2)+3(3a-2)$$
$$(3a-2)(2a+3)$$