

Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Learning Goal 3.2**

Factoring, including the factor theorem and the remainder theorem.

**More Questions - Solutions**

1. Divide each of the following and provide a division statement. Identify any restrictions on the variable.

a.  $P(x) = x^4 - 3x + x^2 - 2x^3 + 4$  by  $x - 1$ .

$$\begin{array}{r|rrrrr}
 1 & 1 & -2 & 1 & -3 & 4 \\
 & \downarrow & & & & \\
 & 1 & -1 & 0 & -3 & \\
 \hline
 & 1 & -1 & 0 & -3 & 1 \\
 \hline
 & x^3 & -x^2 & +0x & -3 & \text{Remainder } 1
 \end{array}$$

$$\begin{array}{r}
 x^3 - x^2 - 3 \\
 \hline
 x-1 \overline{) x^4 - 2x^3 + x^2 - 3x + 4} \\
 \underline{-(x^4 - x^3)} \\
 -x^3 + x^2 \\
 \underline{-(-x^3 + x^2)} \\
 0 - 3x + 4 \\
 \underline{-(-3x + 3)} \\
 1
 \end{array}$$

$$x^4 - 3x + x^2 - 2x^3 + 4 = (x^3 - x^2 - 3)(x - 1) + 1$$

NPV:  $x \neq 1$ b.  $8x^2 + 11 - 6x$  by  $2x - 3$ .

can't use  
synthetic  
division

$$\begin{array}{r}
 4x + 3 \\
 \hline
 2x-3 \overline{) 8x^2 - 6x + 11} \\
 \underline{-(8x^2 - 12x)} \\
 6x + 11 \\
 \underline{-(6x - 9)} \\
 20
 \end{array}$$

$$8x^2 + 11 - 6x = (4x + 3)(2x - 3) + 20$$

NPV:  $x \neq \frac{3}{2}$

c.  $-7x - 6 + x^3$  by  $x + 1$

$$\begin{array}{r|rrrr}
 -1 & 1 & 0 & -7 & -6 \\
 & \downarrow & & & \\
 & 1 & -1 & -6 & 0
 \end{array}$$

$x^2 - x - 6$  Remainder 0

$$\begin{array}{r}
 x^2 - x - 6 \\
 x+1 \overline{) x^3 + 0x^2 - 7x - 6} \\
 \underline{-(x^3 + x^2)} \phantom{-6} \\
 -x^2 - 7x \phantom{-6} \\
 \underline{-(-x^2 - x)} \phantom{-6} \\
 -6x - 6 \\
 \underline{-(-6x - 6)} \\
 0
 \end{array}$$

$$-7x - 6 + x^3 = (x^2 - x - 6)(x + 1)$$

NPVS:  $x \neq -1$

d.  $x^3 + 5x^2 - 2x - 24$  by  $x^2 + 7x + 12$

$$\begin{array}{r}
 x - 2 \\
 x^2 + 7x + 12 \overline{) x^3 + 5x^2 - 2x - 24} \\
 \underline{-(x^3 + 7x^2 + 12x)} \phantom{-24} \\
 -2x^2 - 14x - 24 \\
 \underline{-(-2x^2 - 14x - 24)} \\
 0
 \end{array}$$

$$x^3 + 5x^2 - 2x - 24 = (x - 2)(x^2 + 7x + 12)$$

$$\text{NPVS: } x^2 + 7x + 12 \neq 0$$

$$(x + 3)(x + 4) \neq 0$$

$$\begin{array}{cc}
 \downarrow & \downarrow \\
 x \neq -3 & x \neq -4
 \end{array}$$