Name: ___________________________ Date: ____________________

Learning Goal 3.3	Using more derivative rules.

Explicit Functions/Relations vs **Implicit Functions/Relations**

↳ all we've dealt with

↳ get the dependent variable by itself

$y = \ln(\ln(3x))$

↳ not explicit

$x^2 + y^2 = r^2$

$y^3 - yx + 3 = x^2$

(not solvable for y)

* to take the derivative, implicit differentiation

Example Differentiate $x^2 + y^2 = 25$, then determine the slope of the tangent line at the point $(3, -4)$,

a. Using an explicit function

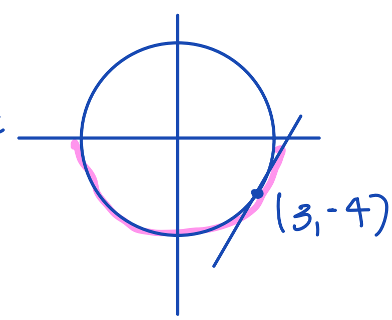
↳ solve for y

$y^2 = 25 - x^2$

$y = \pm\sqrt{25 - x^2}$

$\frac{dy}{dx} = \cancel{-}\frac{1}{\cancel{2}}(25 - x^2)^{-1/2} \times \cancel{-}\cancel{2}x$

$= \frac{x}{\sqrt{25 - x^2}}$



$@\ x = 3,\ \frac{dy}{dx} = \frac{3}{\sqrt{25-9}} = \frac{3}{\sqrt{16}} = \frac{3}{4}$

b. Using an implicit function

$\frac{d}{dx}(x^2 + y^2 = 25)$

$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(25)$

$2x + 2y \times \boxed{\frac{dy}{dx}} = 0$

$2y \times \frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

$(f(x))^2$

$2f(x) \times f'(x)$

$@(3, -4)$

$\frac{dy}{dx} = \frac{-3}{-4} = \frac{3}{4}$

Assignment # 5 – 19, 23 – 29 Quiz Next Day!

PRODUCT RULE.

Example Differentiate $2y^5 + x^2y^2 = x$.

$$\frac{d}{dx}(2y^5) + \frac{d}{dx}(x^2y^2) = \frac{d}{dx}(x)$$

$$2\frac{d}{dx}(y^5) \qquad 2xy^2 + x^2\left(2y \times \frac{dy}{dx}\right) \qquad = 1$$

$$2\left(5y^4 \times \frac{dy}{dx}\right)$$

84% of the question

$$\Rightarrow 10y^4 \times \frac{dy}{dx} + 2xy^2 + 2x^2y \times \frac{dy}{dx} = 1$$

$$\frac{dy}{dx}\left(10y^4 + 2x^2y\right) + 2xy^2 = 1$$

$$\frac{dy}{dx}\left(10y^4 + 2x^2y\right) = 1 - 2xy^2$$

$$\frac{dy}{dx} = \frac{1-2xy^2}{10y^4+2x^2y} \Leftarrow \text{FULL CREDIT}$$

Example Find ${}^{dy}/_{dx}$ if $y = x^{x^2+5x}$.

SOMETIMES CALLED LOGARITHMIC DIFFERENTIATION

$$\ln y = \ln\left(x^{x^2+5x}\right)$$

$$\frac{d}{dx}\left(\ln y = (x^2+5x)\ln x\right)$$

$$y \times \frac{1}{y} \times \frac{dy}{dx} = \left((2x+5)\ln x + (x^2+5x)\frac{1}{x}\right) \times y$$

$$\frac{dy}{dx} = \left[(2x+5)\ln x + (x+5)\right]x^{x^2+5x}$$

Example Find ${}^{dy}/_{dx}$ if $y = (x^2+1)(3x+2)^5$. $\Rightarrow$ COULD USE CHAIN + PRODUCT OR IMPLICIT

$$\ln(xy) = \ln x + \ln y$$

$$\ln x^b = b\ln x$$

$$\ln y = \ln\left((x^2+1)(3x+2)^5\right)$$

$$= \ln(x^2+1) + \ln(3x+2)^5$$

$$\frac{d}{dx}\left(\ln y = \ln(x^2+1) + 5\ln(3x+2)\right)$$

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{1}{x^2+1} \times 2x + \frac{5}{3x+2} \times 3$$

$$y \times \frac{1}{y} \times \frac{dy}{dx} = \left(\frac{2x}{x^2+1} + \frac{15}{3x+2}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{2x}{x^2+1} + \frac{15}{3x+2}\right)(x^2+1)(3x+2)^5$$