Name: ________________________ Date: __________________

Learning Goal 3.3	Using more derivative rules.

More Questions

1. Use implicit differentiation to find the following derivatives.

a. $x^3 + y^3 = 2xy$

$$3x^2 + 3y^2\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y$$
$$3y^2\frac{dy}{dx} - 2x\frac{dy}{dx} = 2y - 3x^2$$
$$\frac{dy}{dx}(3y^2 - 2x) = 2y - 3x^2$$
$$\frac{dy}{dx} = \frac{2y - 3x^2}{3y^2 - 2x}$$

b. $y = \sqrt{\frac{x^2+1}{x^2-1}}$

$$y = \left(\frac{x^2+1}{x^2-1}\right)^{1/2}$$
$$\ln y = \ln\left(\frac{x^2+1}{x^2-1}\right)^{1/2}$$
$$\ln y = \frac{1}{2}\ln\left(\frac{x^2+1}{x^2-1}\right)$$
$$\ln y = \frac{1}{2}(\ln(x^2+1) - \ln(x^2-1))$$
$$\ln y = \frac{1}{2}\ln(x^2+1) - \frac{1}{2}\ln(x^2-1)$$

only now can I start to take the derivative!

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\times\frac{2x}{(x^2+1)} - \frac{1}{2}\times\frac{2x}{(x^2-1)}$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{x}{(x^2+1)} - \frac{x}{(x^2-1)}$$
$$\frac{dy}{dx} = y\left(\frac{x}{(x^2+1)} - \frac{x}{(x^2-1)}\right)$$
$$\frac{dy}{dx} = \sqrt{\frac{x^2+1}{x^2-1}}\left(\frac{x}{(x^2+1)} - \frac{x}{(x^2-1)}\right)$$

c. $\sin y = 3x^2e^{6y+2}$

$$\cos y\frac{dy}{dx} = 6xe^{6y+2} + 3x^2\left(e^{6y+2} \times 6\frac{dy}{dx}\right)$$

$$\cos y\frac{dy}{dx} = 6xe^{6y+2} + 18x^2e^{6y+2}\frac{dy}{dx}$$

$$\cos y\frac{dy}{dx} - 18x^2e^{6y+2}\frac{dy}{dx} = 6xe^{6y+2}$$

$$\frac{dy}{dx}(\cos y - 18x^2e^{6y+2}) = 6xe^{6y+2}$$

$$\frac{dy}{dx} = \frac{6xe^{6y+2}}{\cos y - 18x^2e^{6y+2}}$$

d. $yx^2 + e^y = x$

$$\left(2xy + x^2\frac{dy}{dx}\right) + e^y\frac{dy}{dx} = 1$$

$$2xy + \frac{dy}{dx}(x^2 + e^y) = 1$$

$$\frac{dy}{dx}(x^2 + e^y) = 1 - 2xy$$

$$\frac{dy}{dx} = \frac{1 - 2xy}{x^2 + e^y}$$

e. $2x^3 + x^2y - y^9 = 3x + 4$

$$6x^2 + \left(x^2\frac{dy}{dx} + 2xy\right) - \left(9y^8\frac{dy}{dx}\right) = 3$$

$$6x^2 + 2xy + x^2\frac{dy}{dx} - 9y^8\frac{dy}{dx} = 3$$

$$2x(3x + y) + \frac{dy}{dx}(x^2 - 9y^8) = 3$$

$$\frac{dy}{dx}(x^2 - 9y^8) = 3 - 2x(3x + y)$$

$$\frac{dy}{dx} = \frac{3 - 2x(3x + y)}{x^2 - 9y^8}$$

f. $y^2 + xe^{xy} = 2$

$$2y\frac{dy}{dx} + \left(e^{xy} + xe^{xy}\left(y + x\frac{dy}{dx}\right)\right) = 0$$

$$2y\frac{dy}{dx} + e^{xy} + xye^{xy} + x^2e^{xy}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y + x^2e^{xy}) + e^{xy}(1 + xy) = 0$$

$$\frac{dy}{dx}(2y + x^2e^{xy}) = -e^{xy}(1 + xy)$$

$$\frac{dy}{dx} = \frac{-e^{xy}(1 + xy)}{2y + x^2e^{xy}}$$