

Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Learning Goal 4.1**

Using derivative tests for curve sketching.

**More Questions – Solutions**

1. Find all critical numbers, the interval which  $f$  is increasing or decreasing, and locate any local max or min for.

a.  $f(x) = 2x^3 - 3x^2 - 12x$

$$f'(x) = 6x^2 - 6x - 12$$

$$6x^2 - 6x - 12 = 0$$

$$6(x^2 - x - 2) = 0$$

$$6(x - 2)(x + 1) = 0$$

$$x = -1, 2$$

Interval	$x - 2$	$x + 1$	$f'(x)$	$f(x)$
$x < -1$	-	-	+	inc.
$x = -1$	-	0	0	max
$-1 < x < 2$	-	+	-	dec.
$x = 2$	0	+	0	min
$x > 2$	+	+	+	inc.

**Local Max**

$$\begin{aligned} f(-1) &= 2(-1)^3 - 3(-1)^2 - 12(-1) \\ &= -2 - 3 + 12 \\ &= 7 \end{aligned}$$

$$(-1, 7)$$

**Local Min**

$$\begin{aligned} f(2) &= 2(2)^3 - 3(2)^2 - 12(2) \\ &= 16 - 12 - 24 \\ &= -20 \end{aligned}$$

$$(2, -20)$$

b.  $f(x) = x^4 - 8x^3 + 18x^2$

$$f'(x) = 4x^3 - 24x^2 + 36x$$

$$4x^3 - 24x^2 + 36x = 0$$

$$4x(x^2 - 6x + 9) = 0$$

$$4x(x - 3)^2 = 0$$

$$x = 0, 3$$

Interval	$4x$	$(x - 3)^2$	$f'(x)$	$f(x)$
$x < 0$	-	+	-	dec.
$x = 0$	0	+	0	min
$0 < x < 3$	+	+	+	inc.
$x = 3$	0	+	0	-
$x > 3$	+	+	+	inc.

**Local Min**

$$\begin{aligned} f(0) &= (0)^4 - 8(0)^3 + 18(0)^2 \\ &= 0 \end{aligned}$$

$$(0, 0)$$

**Critical Point**

$$\begin{aligned} f(3) &= (3)^4 - 8(3)^3 + 18(3)^2 \\ &= 81 - 216 + 162 \\ &= 27 \end{aligned}$$

$$(3, 27)$$

c.  $f(x) = x^4 - 8x^2$   
 $f'(x) = 4x^3 - 16x$

$$4x^3 - 16x = 0$$

$$4x(x^2 - 4) = 0$$

$$4x(x + 2)(x - 2) = 0$$

$$x = -2, 0, 2$$

Interval	$4x$	$x + 2$	$x - 2$	$f'(x)$	$f(x)$
$x < -2$	-	-	-	-	dec.
$x = -2$	-	0	-	0	min
$-2 < x < 0$	-	+	-	+	inc.
$x = 0$	0	+	-	0	max
$0 < x < 2$	+	+	-	-	dec.
$x = 2$	+	+	0	0	min
$x > 2$	+	+	+	+	inc.

**Local Min**

$$f(-2) = (-2)^4 - 8(-2)^2$$

$$= 16 - 32$$

$$= -16$$

$$(-2, -16)$$

d.  $f(x) = \frac{2x}{x^2 + 9}$

$$f'(x) = \frac{2(x^2 + 9) - 2x(2x)}{(x^2 + 9)^2}$$

$$= \frac{2(9 - x^2)}{(x^2 + 9)^2}$$

$$2(9 - x^2) = 0$$

$$2(3 + x)(3 - x) = 0$$

$$x = \pm 3$$

**Local Min**

$$f(-3) = \frac{2(-3)}{(-3)^2 + 9}$$

$$= -\frac{6}{18}$$

$$\left(-3, -\frac{1}{3}\right)$$

**Local Max**

$$f(0) = (0)^4 - 8(0)^2$$

$$(0, 0)$$

**Local Min**

$$f(2) = (2)^4 - 8(2)^2$$

$$= 16 - 32$$

$$= -16$$

$$(2, -16)$$

Interval	$3 + x$	$3 - x$	$f'(x)$	$f(x)$
$x < -3$	-	+	-	dec.
$x = -3$	0	+	0	min
$-3 < x < 3$	+	+	+	inc.
$x = 3$	+	0	0	max
$x > 3$	+	-	-	dec.

**Local Max**

$$f(3) = \frac{2(3)}{(3)^2 + 9}$$

$$= \frac{6}{18}$$

$$\left(3, \frac{1}{3}\right)$$

e.  $f(x) = x^3 + 3x^2 + 1$

$$f'(x) = 3x^2 + 6x$$

$$3x^2 + 6x = 0$$

$$3x(x + 2) = 0$$

$$x = -2, 0$$

Interval	$x + 2$	$3x$	$f'(x)$	$f(x)$
$x < -2$	-	-	+	dec.
$x = -2$	0	-	0	min
$-2 < x < 0$	+	-	-	inc.
$x = 0$	+	0	0	max
$x > 0$	+	+	+	dec.

**Local Min**

$$\begin{aligned} f(-2) &= (-2)^3 + 3(-2)^2 + 1 \\ &= -8 + 12 + 1 \\ &= 5 \end{aligned}$$

$(-2, 5)$

**Local Max**

$$\begin{aligned} f(0) &= (0)^3 + 3(0)^2 + 1 \\ &= 1 \end{aligned}$$

$(0, 1)$