Name: _________________________ Date: ___________________

Learning Goal 4.1	Using derivative tests for curve sketching.

More Questions – Solutions

1. For the following functions, find the inflection points, the intervals over which the function is concave up or down, and find any local extrema.
   a. $f(x) = 3x^4 - 4x^3$

$$f'(x) = 12x^3 - 12x^2$$
$$= 12x^2(x-1)$$

$x = 0$ $\qquad$ $x = 1$

$$f''(x) = 36x^2 - 24x$$
$$= 12x(3x-2)$$

$x = 0$ $\qquad$ $x = \frac{2}{3}$

	$12x$	$3x-2$	$f''(x)$	$f(x)$
$x < 0$	$-$	$-$	$+$	up
$x = 0$	0	$-$	0	
$0 < x < \frac{2}{3}$	$+$	$-$	$-$	down
$x = \frac{2}{3}$	$+$	0	0	
$x > \frac{2}{3}$	$+$	$+$	$+$	up

$f''(0) = 0$
Inflection Point

$f''(1) = 12$
Concave Up
Local Minimum

Concave Up: $(-\infty, 0) \cup \left(\frac{2}{3}, \infty\right)$
Concave Down: $\left(0, \frac{2}{3}\right)$

b. $f(x) = x^{2/3}(6-x)^{1/3}$

$$f'(x) = -\frac{1}{3}x^{2/3}(6-x)^{-2/3} + \frac{2}{3}x^{-1/3}(6-x)^{1/3}$$
$$= \frac{1}{3}x^{-1/3}(6-x)^{-2/3}\big(-x+2(6-x)\big)$$
$$= \frac{1}{3}x^{-1/3}(6-x)^{-2/3}(12-3x)$$
$$= x^{-1/3}(6-x)^{-2/3}(4-x)$$
$$= \frac{4-x}{x^{1/3}(6-x)^{2/3}}$$

	$4-x$	$x^{1/3}$	$(6-x)^{2/3}$	$f'(x)$	$f(x)$
$x<0$	$+$	$-$	$+$	$-$	dec
$x=0$	$+$	0	$+$	DNE	
$0<x<4$	$+$	$+$	$+$	$+$	inc
$x=4$	0	$+$	$+$	0	
$4<x<6$	$-$	$+$	$+$	$-$	dec
$x=6$	$-$	$+$	0	DNE	
$x>6$	$-$	$+$	$+$	$-$	dec

$x=0$ Local Minimum　　$x=4$ Local Maximum　　$x=6$ Inflection Point

$$f''(x) = -\frac{1}{3}x^{-4/3}(6-x)^{-2/3}(4-x) + \frac{2}{3}x^{-1/3}(6-x)^{-5/3}(4-x) - x^{-1/3}(6-x)^{-2/3}$$
$$f''(x) = -\frac{1}{3}x^{-4/3}(6-x)^{-5/3}\big((6-x)(4-x) - 2x(4-x) + 3x(6-x)\big)$$
$$f''(x) = -\frac{1}{3}x^{-4/3}(6-x)^{-5/3}(24-10x+x^2-8x+2x^2+18x-3x^2)$$
$$f''(x) = -\frac{1}{3}x^{-4/3}(6-x)^{-5/3}(24)$$
$$f''(x) = -\frac{8}{x^{4/3}(6-x)^{5/3}}$$

$x=0$　　$x=6$

	$-x^{4/3}$	$(6-x)^{5/3}$	$f''(x)$	$f(x)$
$x<0$	$-$	$+$	$-$	down
$x=0$	0	$+$	DNE	
$0<x<6$	$-$	$+$	$-$	down
$x=6$	$-$	0	DNE	
$x>6$	$-$	$-$	$+$	up

Concave Up: $(-\infty,0)\cup(0,6)$

Concave Up: $(6,\infty)$

c. $f(x) = \dfrac{1}{x^2 + 1}$

$$f'(x) = -(x^2+1)^{-2} \times 2x$$
$$= -\frac{2x}{(x^2+1)^2}$$

	$-2x$	$f'(x)$	$f(x)$
$x < 0$	$+$	$+$	inc
$x = 0$	0	0	
$x > 0$	$-$	$-$	dec

$x = 0$

Local Maximum

$$f''(x) = 8x^2(x^2+1)^{-3} - 2(x^2+1)^{-2}$$
$$= 2(x^2+1)^{-3}\left(4x^2 - (x^2+1)\right)$$
$$= \frac{2(3x^2-1)}{(x^2+1)^3}$$

	$3x^2 - 1$	$f''(x)$	$f(x)$
$x < -\sqrt{\frac{1}{3}}$	$+$	$+$	up
$x = -\sqrt{\frac{1}{3}}$	0	0	
$-\sqrt{\frac{1}{3}} < x < \sqrt{\frac{1}{3}}$	$-$	$-$	down
$x = \sqrt{\frac{1}{3}}$	0	0	
$x > \sqrt{\frac{1}{3}}$	$+$	$+$	up

$x = -\sqrt{\frac{1}{3}}$ $\qquad$ $x = \sqrt{\frac{1}{3}}$

Concave Up:

$$\left(-\infty, -\sqrt{\frac{1}{3}}\right) \cup \left(\sqrt{\frac{1}{3}}, \infty\right)$$

Concave Down:

$$\left(-\sqrt{\frac{1}{3}}, -\sqrt{\frac{1}{3}}\right)$$

d. $f\ (x) = x + \dfrac{1}{x}$

$$f'(x) = 1 - \frac{1}{x^2}$$
$$= \frac{x^2 - 1}{x^2}$$

	$x^2 - 1$	x^2	$f'(x)$	$f(x)$
$x < -1$	$+$	$+$	$+$	inc
$x = -1$	0	$+$	0	
$-1 < x < 0$	$-$	$+$	$-$	dec
$x = 0$	$-$	0	DNE	
$0 < x < 1$	$-$	$+$	$-$	dec
$x = 1$	0	$+$	0	
$x > 1$	$+$	$+$	$+$	inc

$x = -1$
Local Maximum

$x = 0$
Inflection Point

$x = 1$
Local Minimum

$$f''(x) = 0 - (-2x^{-3})$$
$$= \frac{2}{x^3}$$

	x^{-3}	$f'(x)$	$f(x)$
$x < 0$	$-$	$-$	down
$x = 0$	0	0	
$x > 0$	$+$	$+$	up

$x = 0$

Concave Up:
$(0, \infty)$

Concave Down:
$(-\infty, 0)$

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e. $f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x$

		$f'(x)$	$f(x)$
$0 < x < \frac{\pi}{4}$	$\cos x > \sin x$	$+$	inc
$x = \frac{\pi}{4}$	$\cos x = \sin x$	0	
$\frac{\pi}{4} < x < \frac{5\pi}{4}$	$\cos x < \sin x$	$-$	dec
$x = \frac{5\pi}{4}$	$\cos x = \sin x$	0	
$\frac{5\pi}{4} < x < 2\pi$	$\cos x > \sin x$	$+$	inc

$x = \frac{\pi}{4} + 2\pi n, n \in \mathbb{Z}$
Local Maximum

$x = \frac{5\pi}{4} + 2\pi n, n \in \mathbb{Z}$
Local Minimum

$f''(x) = -\sin x - \cos x$

		$f''(x)$	$f(x)$
$0 < x < \frac{\pi}{2}$	both +ve	$-$	down
$\frac{\pi}{2} < x < \frac{3\pi}{4}$	$\cos x < \sin x$	$-$	down
$x = \frac{3\pi}{4}$	$-\cos x = \sin x$	0	
$\frac{3\pi}{4} < x < \pi$	$\cos x > \sin x$	$+$	up
$\pi < x < \frac{3\pi}{2}$	both −ve	$+$	up
$x = \frac{7\pi}{4}$	$\cos x = \sin x$	0	
$\frac{7\pi}{4} < x < 2\pi$	$\cos x > \sin x$	$-$	down

$x = \frac{3\pi}{4} + 2\pi n, n \in \mathbb{Z}$ $x = \frac{7\pi}{4} + 2\pi n, n \in \mathbb{Z}$

Concave Up:
$\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right) + 2\pi n, n \in \mathbb{Z}$

Concave Down:
$\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right) + 2\pi n, n \in \mathbb{Z}$

Assignment # 11 – 19 , 33 – 37 Quiz Next Day!