

Name: _____

Date: _____

Learning Goal 6.1Simplifying and applying operations to rational
expressions, identifying any non-permissible values.**Recall** Multiplying and Dividing Fractions

$$\begin{array}{l}
 \text{Mult.} \quad = \frac{5}{8} \times \frac{2}{35} \\
 \text{then} \quad = \frac{10 \div 10}{280 \div 10} \\
 \text{Simp.} \quad = \frac{1}{28}
 \end{array}
 \quad
 \begin{array}{l}
 \text{Simp.} \quad = \frac{1}{4} \times \frac{5}{8} \times \frac{1}{35} \\
 \text{then} \quad = \frac{1}{28}
 \end{array}
 \quad
 \begin{array}{l}
 \frac{7}{9} \div \frac{35}{6} \\
 = \frac{1}{3} \times \frac{6^2}{35} \\
 = \frac{1}{3} \times \frac{2}{5} \\
 = \frac{2}{15}
 \end{array}$$

Extend to rational expressions. Simplify and state the non-permissible values.

a. $\left(\frac{4x^2}{3xy}\right)\left(\frac{y^2}{8x}\right)^2$ **NPV:** $x \neq 0$
 $y \neq 0$

$$\begin{aligned}
 &= \frac{x^2 y^2}{6x^2 y} \\
 &= \frac{y^2}{6y} \\
 &= \frac{y}{6}
 \end{aligned}$$

c. $\frac{x+3}{2} \times \frac{x+1}{4}$ **NPV:** none

$$\begin{aligned}
 &= \frac{(x+3)(x+1)}{8} \\
 &= \frac{x^2 + 4x + 3}{8}
 \end{aligned}$$

b. $\frac{10ac^3}{3b^2} \div \frac{2b^5c^4}{15ab^2}$ **NPV:** $a \neq 0$
 $b \neq 0$
 $c \neq 0$

$$\begin{aligned}
 &= \frac{5}{1} \times \frac{15ab^2}{3b^2} \\
 &= \frac{25a^2b^2c^3}{b^7c^4} \\
 &= 25a^2b^{-5}c^{-1} \\
 &= \frac{25a^2}{b^5c}
 \end{aligned}$$

d. $\frac{x-3}{x^2-9} \div \frac{x}{x+3}$ **NPV:** $x+3 \neq 0$
 $x = -3$
 $x-3 \neq 0$
 $x \neq 3$
 $x \neq 0$

$$\begin{aligned}
 &= \frac{(x-3)}{(x+3)(x-3)} \div \frac{x}{x+3} \\
 &= \frac{1}{(x+3)(x-3)} \times \frac{x+3}{x} \\
 &= \frac{1}{x}
 \end{aligned}$$

$$\text{e. } \frac{a^2 - a - 12}{a^2 - 9} \times \frac{a^2 - 4a + 3}{a^2 - 4a}$$

$$\begin{aligned} & a^2 - a - 12 \\ & = a^2 - 4a + 3a - 12 \\ & = a(a-4) + 3(a-4) \\ & = (a-4)(a+3) \end{aligned}$$

$$\begin{aligned} & = \frac{(a-4)(a+3)}{(a+3)(a-3)} \times \frac{(a-3)(a-1)}{a(a-4)} \\ & = \frac{a-1}{a} \end{aligned}$$

$$\text{f. } \frac{x^2 - 4}{x^2 - 4x} \div \frac{x^2 + x - 6}{x^2 + x - 20}$$

$$\begin{aligned} x^2 - 4 &= (x+2)(x-2) \\ x^2 - 4x &= x(x-4) \\ x^2 + x - 6 &= (x+3)(x-2) \\ x^2 + x - 20 &= (x+5)(x-4) \end{aligned}$$

NPV:

$$\begin{aligned} a+3 &\neq 0 \\ a &\neq -3 \\ a-3 &\neq 0 \\ a &= 3 \\ a &\neq 0 \\ a-4 &\neq 0 \\ a &\neq 4 \end{aligned}$$

$$= \frac{(x+2)(x-2)}{x(x-4)} \div \frac{(x+3)(x-2)}{(x+5)(x-4)}$$

$$\begin{aligned} \text{NPV: } x &\neq 0 \\ x-4 &\neq 0 \\ x &\neq 4 \\ x+5 &\neq 0 \\ x-5 &\neq 0 \\ x &\neq -5 \end{aligned}$$

$$= \frac{(x+2)(x-2)}{x(x-4)} \times \frac{(x+5)(x-4)}{(x+3)(x-2)}$$

$$= \frac{(x+2)(x+5)}{x(x+3)}$$

Example Write a polynomial A so that the expression simplifies to -1 .

$$\frac{3n^2 + 2n - 8}{n^2 + 4n + 4} \times \frac{A}{3n^2 - n - 4} = -1$$

Step 1:
Factor!

$$\begin{aligned} 3n^2 + 2n - 8 & \\ = 3n^2 + 6n - 4n - 8 & \\ = 3n(n+2) - 4(n+2) & \\ = (n+2)(3n-4) & \end{aligned}$$

$$\begin{aligned} n^2 + 4n + 4 & \\ = (n+2)(n+2) & \end{aligned}$$

$$\begin{aligned} 3n^2 - n - 4 & \\ = 3n^2 + 3n - 4n - 4 & \\ = 3n(n+1) - 4(n+1) & \\ = (n+1)(3n-4) & \end{aligned}$$

$$\frac{(n+2)(3n-4)}{(n+2)(n+2)} \times \frac{A}{(n+1)(3n-4)} = -1$$

$$\frac{A}{(n+2)(n+1)} = -1$$

$$A = -(n+2)(n+1)$$

CHECK

$$-\frac{(n+2)(n+1)}{(n+2)(n+1)} = \frac{-1}{1} = -1$$