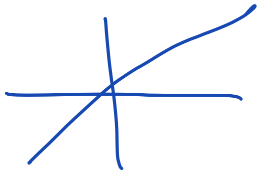
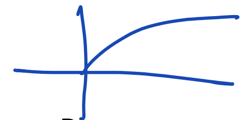




Name: ___________________________ Date: ____________________

Learning Goal 6.1	Using identities to reduce complexity in expressions and solve equations.

Sum and Difference Identities

$$\cos(A+B) = \cos A\cos B - \sin A\sin B \qquad \sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B \qquad \sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Example Verify the identity $\sin(A-B) = \sin A\cos B - \cos A\sin B$ numerically, without a calculator, for

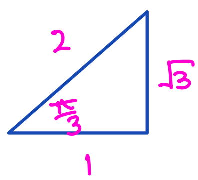
$$\measuredangle A = \frac{\pi}{3} \quad \text{and} \quad \measuredangle B = \frac{\pi}{4}.$$

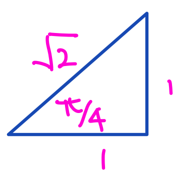
$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \sin\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$= \sin\left(\frac{\pi}{12}\right)$

$\doteq 0.2588$





$\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$

$\doteq 0.2588$

Example Express the following as a trigonometric function of a single angle.

$$\sin(\pi)\cos\left(\frac{\pi}{5}\right) - \cos(\pi)\sin\left(\frac{\pi}{5}\right)$$

A B A B

$= \sin\left(\pi - \frac{\pi}{5}\right)$

$= \sin\left(\frac{5\pi}{5} - \frac{\pi}{5}\right)$

$= \sin\left(\frac{4\pi}{5}\right)$

Example Simplify and then give an exact value for each expression.

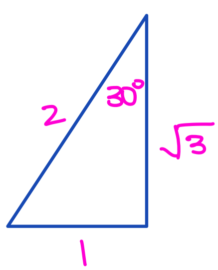
a. $\cos 25^\circ \cos 5^\circ - \sin 25^\circ \sin 5^\circ$

A B A B

$= \cos(25^\circ + 5^\circ)$

$= \cos(30^\circ)$

$= \frac{\sqrt{3}}{2}$



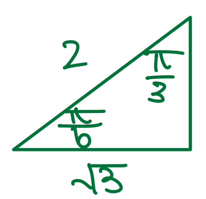
b. $\sin\frac{\pi}{3}\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\sin\frac{\pi}{6}$

A B A B

$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{2\pi}{6} + \frac{\pi}{6}\right)$

$= \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

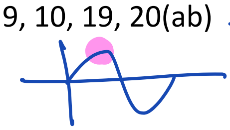


$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} + \frac{1}{4}$

$= 1$

Assignment p. 306 # 1(abd), 2(abd), 6, 7, 8(ace), 9, 10, 19, 20(ab) Quiz Next Day!

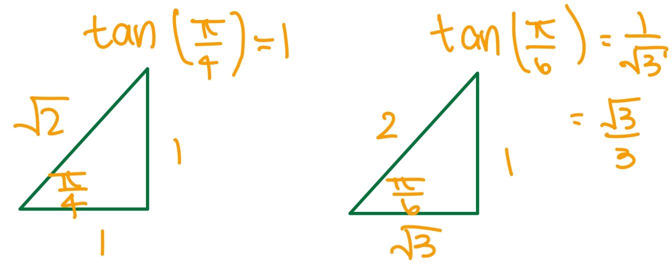


Example Simplify $\sin(x-\pi)$.

$= \sin x \cos \pi - \cos x \sin \pi$

$= \sin x(-1) - \cos x(0)$

$= -\sin x$



Example Determine the exact value of $\tan\left(\frac{\pi}{12}\right)$.

$\frac{\pi}{6} = \frac{2\pi}{12}$

$\frac{\pi}{4} = \frac{3\pi}{12}$

$\frac{\pi}{3} = \frac{4\pi}{12}$

$\frac{\pi}{2} = \frac{6\pi}{12}$

$\pi = \frac{12\pi}{12}$

$= \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \dfrac{\tan\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{6}\right)}{1 + \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{6}\right)}$

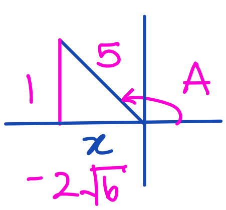
$= \dfrac{1 - \frac{\sqrt{3}}{3}}{1 + (1)\left(\frac{\sqrt{3}}{3}\right)} = \dfrac{\left(\frac{3-\sqrt{3}}{3}\right)}{\left(\frac{3+\sqrt{3}}{3}\right)} = \frac{3-\sqrt{3}}{3} \times \frac{3}{3+\sqrt{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}}$

$= \dfrac{9 - 3\sqrt{3} - 3\sqrt{3} + 3}{9 - 3\sqrt{3} + 3\sqrt{3} - 3}$

$= \dfrac{12 - 6\sqrt{3}}{6}$

$= 2 - \sqrt{3}$

Example If $\sin A = \frac{1}{5}$ and $\cos B = \frac{4}{7}$, $\measuredangle A$ is in QII and $\measuredangle B$ is in Q1, find an exact value for $\cos(A+B)$.



$x^2 + y^2 = r^2$

$x^2 + (1)^2 = (5)^2$

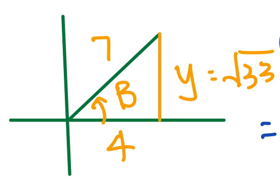
$x^2 + 1 = 25$

$-1 \quad -1$

$\sqrt{x^2} = \sqrt{24}$

$x = \pm\sqrt{24}$

$= \pm 2\sqrt{6}$



$x^2 + y^2 = r^2$

$(4)^2 + y^2 = (7)^2$

$16 + y^2 = 49$

$-16 \quad -16$

$\sqrt{y^2} = \sqrt{33}$

$y = \pm\sqrt{33}$

$\cos(A+B) =$

$= \cos A \cos B - \sin A \sin B$

$= \left(-\frac{2\sqrt{6}}{5}\right)\left(\frac{4}{7}\right) - \left(\frac{1}{5}\right)\left(\frac{\sqrt{33}}{7}\right)$

$= -\frac{8\sqrt{6}}{35} - \frac{\sqrt{33}}{35}$

$= \dfrac{-8\sqrt{6} - \sqrt{33}}{35}$

Assignment p. 306 # 1(abd), 2(abd), 6, 7, 8(ace), 9, 10, 19, 20(ab) Quiz Next Day!