Name: ________________________ Date: ___________________

Learning Goal 6.1	Using identities to reduce complexity in expressions and solve equations.

More Questions – Solutions

Sum and Difference Identities

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

1. Verify the identity $\sin(A+B) = \sin A \cos B + \cos A \sin B$ numerically, without a calculator, for

$$\measuredangle A = \frac{\pi}{2} \quad \text{and} \quad \measuredangle B = \frac{\pi}{6}.$$

A B

$\sin\left(\frac{\pi}{2}\right) = 1$ $\cos\left(\frac{\pi}{2}\right) = 0$ $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\sin(A+B)$	$\sin A \cos B + \cos A \sin B$
$= \sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$	$= (1)\left(\frac{\sqrt{3}}{2}\right) + (0)\left(\frac{1}{2}\right)$
$= \sin\left(\frac{2\pi}{3}\right)$	$= \frac{\sqrt{3}}{2}$
$= \sin\left(\frac{\pi}{3}\right)$	
$= \frac{\sqrt{3}}{2}$	

2. Express the following as a trigonometric function of a single angle.

$$\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{8}\right) - \cos\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{8}\right)$$
$$= \sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right)$$
$$= \sin\left(\frac{3\pi}{8}\right)$$

3. Simplify and then give an exact value for each expression.

a. $\cos 108° \cos 18° + \sin 108° \sin 18°$

$= \cos(108° - 18°)$

$= \cos(90°)$

$= 0$

b. $\dfrac{\tan\left(\frac{5\pi}{6}\right) - \tan\left(\frac{\pi}{3}\right)}{1 + \tan\left(\frac{5\pi}{6}\right)\tan\left(\frac{\pi}{3}\right)}$

$= \tan\left(\frac{5\pi}{6} - \frac{\pi}{3}\right)$

$= \tan\left(\frac{\pi}{2}\right)$

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4. Simplify $\cos(x + \pi)$.

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

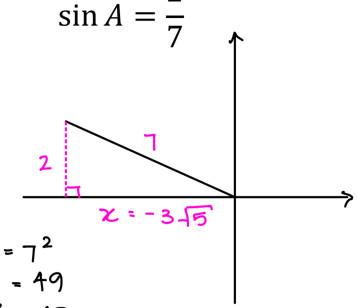
$\cos(x + \pi) = \cos x \cos \pi - \sin x \sin \pi$

$\cos(x + \pi) = \cos x\,(-1) - \sin x\,(0)$

$\cos(x + \pi) = -\cos x$

5. If $\sin A = \frac{2}{7}$ and $\cos B = \frac{3}{5}$, $\measuredangle A$ is in QII and $\measuredangle B$ is in QIII, find an exact value for $\cos(A - B)$.

$\sin A = \frac{2}{7}$



$2^2 + x^2 = 7^2$

$4 + x^2 = 49$

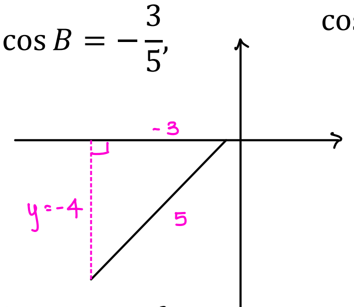
$x^2 = 45$

$x = -\sqrt{45}$

$= -3\sqrt{5}$

$\cos A = -\frac{\sqrt{45}}{7} = -\frac{3\sqrt{5}}{7}$

$\cos B = -\frac{3}{5},$



$(-3)^2 + y^2 = 5^2$

$9 + y^2 = 25$

$y^2 = 16$

$y = -4$

$\sin B = -\frac{4}{5}$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$= \left(-\frac{3\sqrt{5}}{7}\right)\left(-\frac{3}{5}\right) + \left(\frac{2}{7}\right)\left(-\frac{4}{5}\right)$

$= \frac{9\sqrt{5}}{35} - \frac{8}{35}$

$= \frac{-8 + 9\sqrt{5}}{35}$

6. Determine the exact value of $\tan\left(\frac{\pi}{12}\right)$.

$$\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{3\pi}{12} - \frac{2\pi}{12}\right)$$
$$= \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$

$$A = \frac{\pi}{4} \qquad B = \frac{\pi}{6}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\tan\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{6}\right)}{1 + \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{6}\right)}$$

$$= \frac{(1) - \left(\frac{\sqrt{3}}{3}\right)}{1 + (1)\left(\frac{\sqrt{3}}{3}\right)}$$

$$= \frac{(3 - \sqrt{3})/3}{(3 + \sqrt{3})/3}$$

$$= \frac{(3 - \sqrt{3})}{3} \times \frac{3}{(3 + \sqrt{3})}$$

$$= \frac{(3 - \sqrt{3})}{(3 + \sqrt{3})} \times \frac{(3 - \sqrt{3})}{(3 - \sqrt{3})}$$

$$= \frac{(3 - \sqrt{3})^2}{6}$$

$$= \frac{(3 - \sqrt{3})^2}{6}$$

$$= \frac{12 - 2\sqrt{3}}{6}$$

$$= \frac{6 - \sqrt{3}}{3}$$