Name: _____ Date: _____

Learning Goal 8.1	Solving exponential and logarithmic equations with same base and with different bases, including base e.

Example Recall that

$b^x = b$
$x = 1$

$$\log_b b = 1$$

and since logarithms and exponentials are opposite operations,

$$\log_b b^c = c \times \log_b b$$

Power Law

$$\log_b(x^a) = a \log_b x$$

Example Recall that $c^x c^y = c^{x+y}$, and let's extend that to logarithms.

THIS DOES NOT MEAN
$\log_b(x+y) \neq \log_b x \log_b y$

Product Law

$$\log_b x + \log_b y = \log_b(xy)$$

Example Recall that ${}^{c^x}/_{c^y} = c^{x-y}$, and let's extend that to logarithms.

$$\log_b\left(\frac{x}{y}\right) = \log_b(xy^{-1}) = \log_b x + \log_b y^{-1}$$

PRODUCT

$$= \log_b x + (-1)\log_b y$$ POWER.

IT IS NOT TRUE
$\log_b(x-y) \neq \frac{\log_b x}{\log_b y}$

Quotient Law

$$\log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$$

Example Write each expression in terms of individual logarithms.

a. $\log_6 \frac{xy}{z}$ $= \log_6\left(\frac{x}{z} \times y\right) = \log_6\left(x \times \frac{y}{z}\right)$

$= (\log_6 x + \log_6 y) - \log_6 z$

$= (\log_6 x - \log_6 z) + \log_6 y$

b. $\log_7 \sqrt{xy}$ $= \log_7 (xy)^{1/2}$ ↘ power

$= \frac{1}{2}\log_7 (xy)$ ↘ product

$= \frac{1}{2}(\log_7 x + \log_7 y)$

$= \frac{1}{2}\log_7 x + \frac{1}{2}\log_7 y$

Example Simplify using logarithm laws.

a. $\log 25 + \log 4$ PRODUCT

$= \log(25 \times 4)$

$= \log_{10}(100)$

$= 2$

b. $\log_5 50 - \log_5 0.4$ QUOTIENT

$= \log_5\left(\frac{50}{0.4}\right)$

$= \log_6 (125)$

$= 3$

$\frac{50}{\left(\frac{2}{5}\right)} = 50 \times \frac{5}{2}$

$= 25 \times 5$

$= 125$

c. $3\log_3 6 - 4\log_3 2 + \frac{1}{2}\log_3 4$

$= \log_3 6^3 - \log_3 2^4 + \log_3 4^{1/2}$ ↘ power

$= \log_3 216 - \log_3 16 + \log_3 2$ ↘ quotient

$= \log_3\left(\frac{216}{16}\right) + \log_3 2$

$= \log_3\left(\frac{27}{2}\right) + \log_3 2$ PRODUCT ↷ $= \log_3\left(\frac{27}{2} \times 2\right) = \log_3 (27) = 3$

Example If $\log_2 5 = p$, express each logarithm in terms of p.

a. $\log_2 20$

$= \log_2 (5 \times 4)$ ↘ PRODUCT RULE.

$= \log_2 5 + \log_2 4$

$= p + 2$

b. $\log_2\left(\frac{\sqrt[3]{5}}{2}\right)$

$= \log_2\left(\frac{5^{1/3}}{2}\right)$

$= \log_2 5^{1/3} - \log_2 2$

$= \frac{1}{3}\log_2 5 - \log_2 2$

$= \frac{1}{3}p - 1$

$= \frac{p-3}{3}$