

Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Learning Goal 2.2** Using trigonometric ratios and solving simple trigonometric equations.

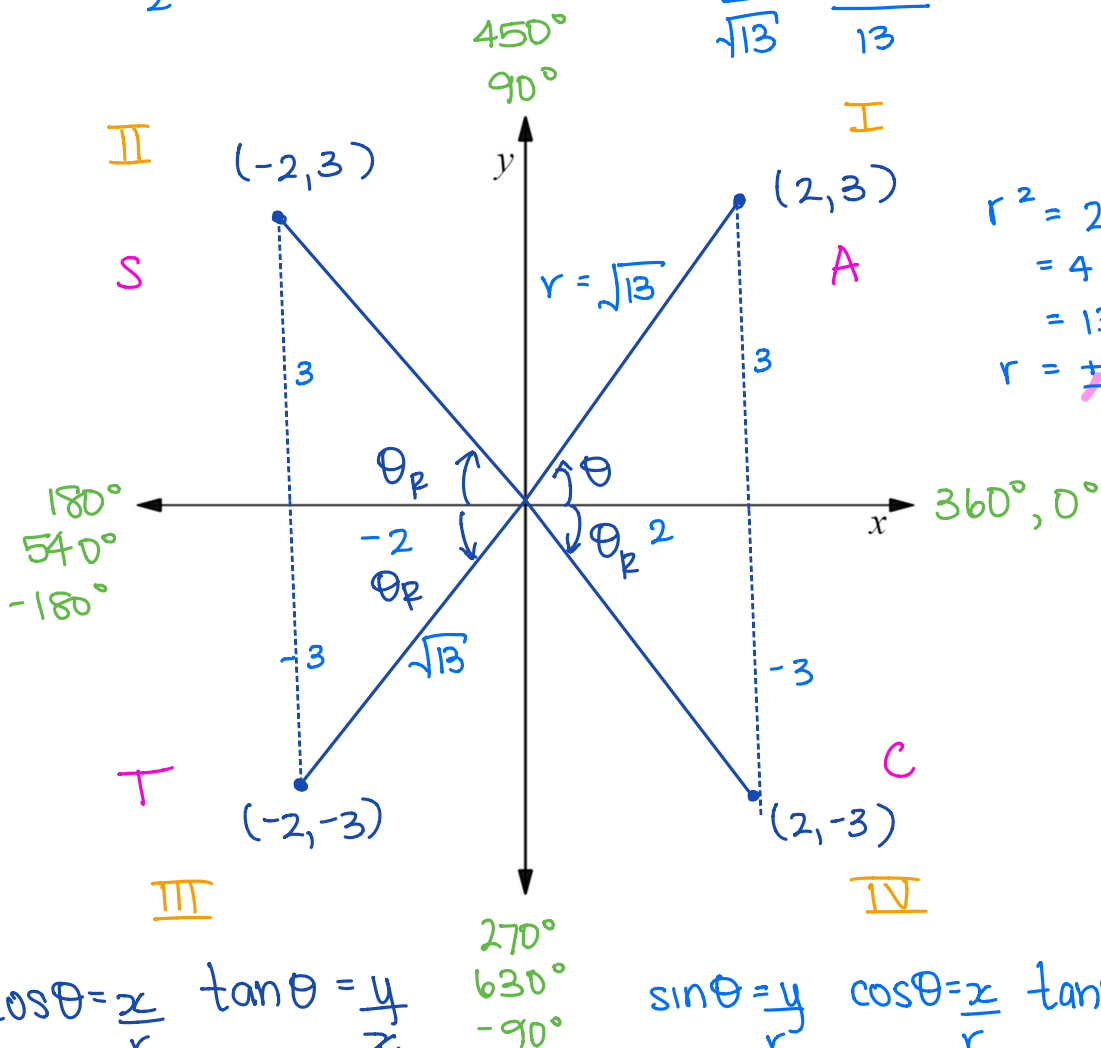
$$\begin{aligned} \cos \theta &= \frac{x}{r} \\ &= \frac{-2}{\sqrt{13}} \\ &= \frac{-2\sqrt{13}}{13} \end{aligned}$$

$$\begin{aligned} \tan \theta &= \frac{y}{x} \\ &= \frac{3}{-2} \end{aligned}$$

$$\begin{aligned} \cos \theta &= \frac{\text{adj}}{\text{hyp}} \\ &= \frac{x}{r} \\ &= \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13} \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{\text{opp}}{\text{hyp}} \\ &= \frac{y}{r} \\ &= \frac{3}{\sqrt{13}} \end{aligned}$$

$$\begin{aligned} \tan \theta &= \frac{\text{opp}}{\text{adj}} \\ &= \frac{y}{x} \\ &= \frac{3}{2} \end{aligned}$$



$$\begin{aligned} r^2 &= 2^2 + 3^2 \\ &= 4 + 9 \\ &= 13 \\ r &= \pm \sqrt{13} \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{y}{r} \\ &= \frac{-3}{\sqrt{13}} \\ &= \frac{-3\sqrt{13}}{13} \end{aligned}$$

$$\begin{aligned} \cos \theta &= \frac{x}{r} \\ &= \frac{-2}{\sqrt{13}} \\ &= \frac{-2\sqrt{13}}{13} \end{aligned}$$

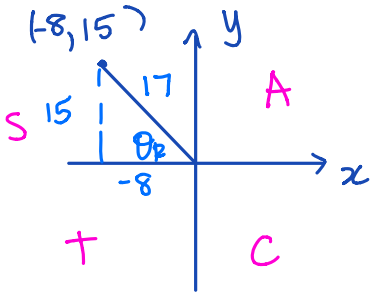
$$\begin{aligned} \tan \theta &= \frac{y}{x} \\ &= \frac{-3}{-2} \\ &= \frac{3}{2} \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{y}{r} \\ &= \frac{-3}{\sqrt{13}} \\ &= \frac{-3\sqrt{13}}{13} \end{aligned}$$

$$\begin{aligned} \cos \theta &= \frac{x}{r} \\ &= \frac{2}{\sqrt{13}} \\ &= \frac{2\sqrt{13}}{13} \end{aligned}$$

$$\begin{aligned} \tan \theta &= \frac{y}{x} \\ &= \frac{-3}{2} \end{aligned}$$

**Example** The point  $P(-8, 15)$  lies on the terminal arm of an angle  $\theta$ , in standard position. Determine the exact trigonometric ratios for  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$ .



$$\tan \theta = \tan \theta_P = -\frac{15}{8}$$

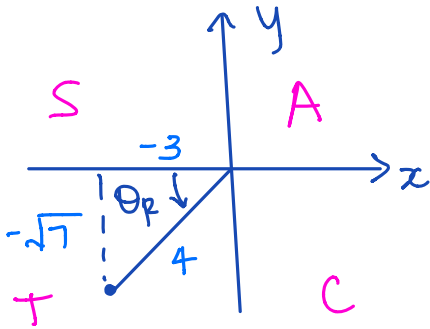
$$\sin \theta = \sin \theta_P = \frac{15}{17}$$

$$\begin{aligned} r^2 &= (-8)^2 + 15^2 \\ &= 64 + 225 \\ &= 289 \end{aligned}$$

$$\cos \theta = \cos \theta_P = \frac{-8}{17}$$

$$r = \sqrt{289} = 17$$

**Example** Suppose  $\theta$  is an angle in standard position with terminal arm in quadrant III, and  $\cos \theta = -3/4$ . What are the exact values of  $\sin \theta$  and  $\tan \theta$ ?



$$\begin{aligned} \cos \theta &= \frac{\text{adj}}{\text{hyp}} \\ &= \frac{x}{r} \\ &= \frac{-3}{4} \end{aligned}$$

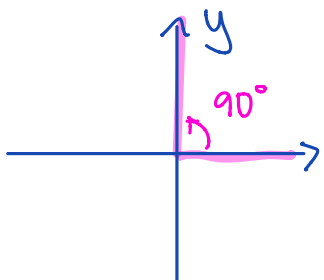
$$\begin{aligned} r^2 &= x^2 + y^2 \\ (4)^2 &= (-3)^2 + y^2 \\ 16 &= 9 + y^2 \\ 7 &= y^2 \\ y &= \pm\sqrt{7} \end{aligned}$$

$$\begin{aligned} \sin \theta &= \sin \theta_P \\ &= \frac{y}{r} \\ &= \frac{-\sqrt{7}}{4} \end{aligned}$$

$$\begin{aligned} \tan \theta &= \tan \theta_P \\ &= \frac{y}{x} \\ &= \frac{-\sqrt{7}}{-3} \\ &= \frac{\sqrt{7}}{3} \end{aligned}$$

$$\Rightarrow x = -3, r = 4$$

**Example** Determine the values of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  when the terminal arm of quadrantal angle  $\theta$  coincides with the positive  $y$ -axis.



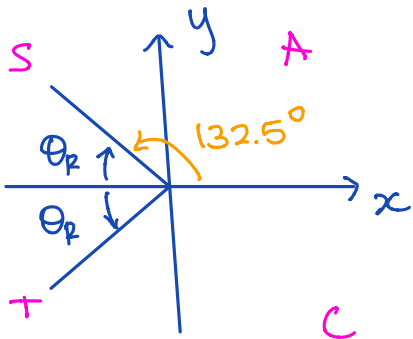
$$\begin{aligned} r^2 &= x^2 + y^2 \\ r^2 &= y^2 \\ r &= y \\ x &= 0 \end{aligned}$$

$$\sin \theta = \frac{y}{r} = 1$$

$$\cos \theta = \frac{x}{r} = \frac{0}{r} = 0$$

$$\tan \theta = \frac{y}{x} = \frac{r}{0} = \text{DNE}$$

**Example** Given  $\cos \theta = -0.6753$  where  $0^\circ \leq \theta < 360^\circ$ , determine the measure of  $\theta$  to the nearest tenth of a degree.



$$\cos \theta = -0.6753$$

$$\begin{aligned} \theta_1 &= \cos^{-1}(-0.6753) \\ &= 132.5^\circ \Rightarrow \text{II} \end{aligned}$$

$$\begin{aligned} \theta_2 &= 180 + 47.5 \\ &= 227.5^\circ \end{aligned}$$

$$\begin{aligned} \theta_R &= 180 - 132.5^\circ \\ &= 47.5^\circ \end{aligned}$$