Name: ________________________ Date: __________________

Learning Goal 3.7	Creating confidence in word problems.

Related Rates

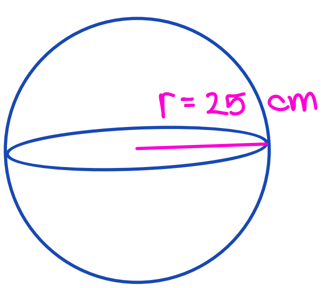
- Given a function $y=f(x)$ where both x and y are changing (usually with respect to time)
- Need implicit differentiation to solve.

General Steps: watch units!

1. Draw a diagram if possible and write down the givens using appropriate notations.
2. Write down which rate you want to solve for.
3. Write an equation that relates the variables of the problem. If there are more than two variables, try to eliminate one by substitution and/or the geometric property of the problem.
4. Use implicit differentiation to differentiate both sides of the equation with respect to t.
5. Substitute the given information into the resulting equation and solve for the unknown rate.

Example Air is being pumped into a spherical balloon so that its volume increases at a rate of $100\ \mathrm{cm^3/s}$. How fast is the radius of the balloon increasing when the diameter is 50 cm?

1.



$$\frac{dV}{dt} = 100\ \mathrm{cm^3/s}$$

2. $$\frac{dr}{dt} = ?$$

3. $$V = \frac{4}{3}\pi r^3$$ ← SHOULD KNOW ...

4. $$\frac{dV}{dt} = \frac{4}{3}\pi \times 3r^2 \times \frac{dr}{dt}$$

5. $$100 = \frac{4}{3}\pi \times 3(25)^2 \times \frac{dr}{dt}$$

$$\cancel{100}^{\,\cancel{25}\,1} \times \frac{\cancel{3}^{\,1}}{\cancel{4}\pi} \times \frac{1}{\cancel{3}_{\,1}(25)^{\cancel{2}}} = \frac{dr}{dt}$$

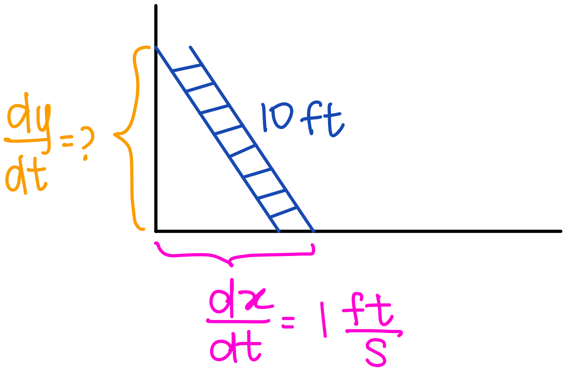
$$\frac{dr}{dt} = \frac{1}{25\pi}\ \mathrm{cm/s}$$

radius units

time units

Assignment # 1 – 19 Quiz Next Day!

Example A 10 ft ladder rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 $^{\text{ft}}/_{\text{s}}$, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?



$x = 6$ ft

PYTHAGORAS!

$$x^2 + y^2 = 10^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(6)(1) + 2y\frac{dy}{dt} = 0$$

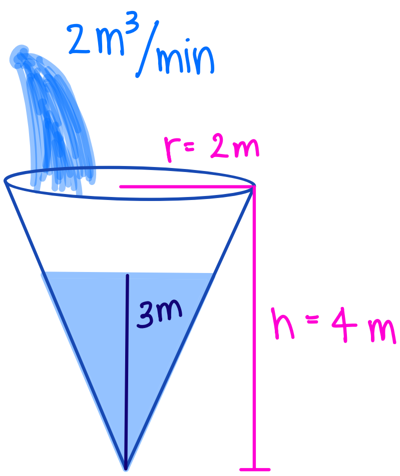
too many unkowns...

$$x^2 + y^2 = 10^2$$
$$6^2 + y^2 = 100$$
$$y^2 = 64$$
$$y = 8$$

$$12 + 2(8)\frac{dy}{dt} = 0$$

$$16\frac{dy}{dt} = -12 \Rightarrow \frac{dy}{dt} = -\frac{12}{16} = -\frac{3}{4}\ \text{ft/s}$$

Example A water tank has the shape of an inverted circular cone with base radius of 2 m and a height of 4 m. If the water is being poured into the tank at a rate of 2 $^{\text{m}^3}/_{\text{min}}$, find the rate at which the water level is rising when the water is 3 m deep.



$\frac{dh}{dt} = ?$

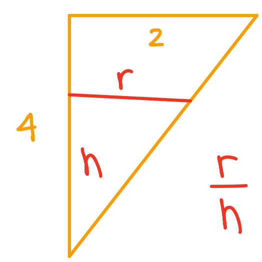
VOLUME OF A CONE?

$$V = \frac{1}{3}\pi r^2 h$$

BOTH r AND h CHANGE AS THE WATER LEVEL RISES

$$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$

$$= \frac{1}{3}\pi\frac{h^3}{4}$$



$$\frac{r}{h} = \frac{2}{4} = \frac{1}{2}$$

$$r = \frac{h}{2}$$

$$\frac{dV}{dt} = \frac{1}{3}\pi \times \frac{3}{4}h^2 \times \frac{dh}{dt}$$

$$2 = \frac{1}{\cancel{3}}\pi \times \frac{\cancel{3}}{4}(3)^2 \times \frac{dh}{dt}$$

$$2 \times \frac{1}{\pi} \times \frac{4}{3^2} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{8}{9\pi}\ \text{m/min}$$

Assignment # 1 – 19 Quiz Next Day!

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{2}{4} = \frac{r}{3}$$

$$\frac{dV}{dt} = \frac{1}{3}\pi r^2 \frac{dh}{dt} \qquad r = \frac{3}{2}m$$

$$2 = \frac{1}{3}\pi \left(\frac{3}{2}\right)^2 \frac{dh}{dt}$$

$$2 \times \frac{3}{\pi} \times \frac{4}{9} = \frac{dh}{dt}$$