Name: ______________________ Date: __________________

Learning Goal 3.1	Using all basic derivative rules.

Example Differentiate the following in two ways – multiplying first and multiplying during.

a. $y = 3x^2(4x)$

$= 12x^3$ $\frac{dy}{dx} = 12 \times 3x^2$ $= 36x^2$!!!	$\frac{dy}{dx} = 6x \times 4$ $= 24x$

b. $y = \dfrac{6x^5}{2x^3}$

The Product Rule

$$m(x) = f(x)g'(x) + f'(x)g(x)$$

where $m(x) = f(x)g(x)$

Example Find the derivative using the product rule.

a. $y = \underbrace{3x^2}_{f}\underbrace{(4x)}_{g}$

$f'(x) = 6x$ $g'(x) = 4$

$$\frac{dy}{dx} = 4(3x^2) + 6x(4x)$$
$$= 12x^2 + 24x^2$$
$$= 36x^2$$

b. $m(x) = \underbrace{\sqrt[3]{x^2}}_{f}\underbrace{(2x - x^2)}_{g}$

$f(x) = x^{2/3}$ $g'(x) = 2 - 2x$

$f'(x) = \frac{2}{3}x^{-1/3}$

$= \dfrac{2}{3\sqrt[3]{x}}$

$= \dfrac{2\sqrt[3]{x^2}}{3x}$

$$m'(x) = \sqrt[3]{x^2}(2-2x) + \frac{2\sqrt[3]{x^2}}{3x}(2x - x^2)$$
$$= 2\sqrt[3]{x^2} - 2x\sqrt[3]{x^2} + \frac{4\sqrt[3]{x^2}}{3} - \frac{2x\sqrt[3]{x^2}}{3}$$

$$= 2\sqrt[3]{x^2}\left(1 - x + \frac{2}{3} - \frac{x}{3}\right)$$

$= \frac{2\sqrt[3]{x^2}}{3}(3-3x+2-x)$

$= \frac{2\sqrt[3]{x^2}}{3}(5-4x)$

The Quotient Rule

if $m(x) = \frac{f(x)}{g(x)}$ then $m'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$ $\frac{\text{LODEHI} - \text{HIDELO}}{\text{LO}^2}$

Example Find the derivative using the quotient rule.

a. $y = \frac{6x^5}{2x^3} = 3x^2$ $\frac{dy}{dx} = 6x$

$f'(x) = 30x^4$ $g'(x) = 6x^2$

$\frac{dy}{dx} = \frac{(2x^3)(30x^4) - (6x^5)(6x^2)}{(2x^3)^2}$

$= \frac{60x^7 - 36x^7}{4x^6}$

$= \frac{24x^7}{4x^6}$

$= 6x$

b. $p(t) = \frac{3t+9}{2-t}$

$f'(t) = 3$
$g'(t) = -1$

$p'(t) = \frac{(2-t)(3) - (3t+9)(-1)}{(2-t)^2}$

$= \frac{6 \cancel{-3t} + \cancel{3t} + 9}{(2-t)^2}$

$= \frac{15}{(2-t)^2}$

Example Suppose that the volume of air in a balloon at time t seconds, is given by the formula

$$v(t) = \frac{6\sqrt{t}}{4t+1}$$

Determine if the balloon is being filled with or is losing air when $t = 8$ seconds.

$f'(t) = \frac{3}{\sqrt{t}} = \frac{3\sqrt{t}}{t}$ $g'(t) = 4$

$v'(t) = \frac{(4t+1)\left(\frac{3\sqrt{t}}{t}\right) - (6\sqrt{t})(4)}{(4t+1)^2}$

$(6\sqrt{8})(4)$
$(12\sqrt{2})(4)$
$48\sqrt{2}$

$v'(8) = \frac{(33)\left(\frac{\cancel{6}^{3}\sqrt{2}}{\cancel{8}_{4}}\right) - 48\sqrt{2}}{(33)^2}$

$= \frac{\left(\frac{99}{4} - 48\right)\sqrt{2}}{33^2} < 0$ $\Rightarrow$ the balloon is deflating.

$$m(x) = f(x) \times g(x)$$

$$m'(x) = \lim_{h \to 0} \frac{m(x+h) - m(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \left[\frac{f(x+h)g(x+h) - f(x+h)g(x)}{h} \right] \left[+ f(x+h)g(x) - f(x)g(x) \right]$$

$$= \lim_{h \to 0} f(x+h)\left(\frac{g(x+h) - g(x)}{h}\right) + g(x)\left(\frac{f(x+h) - f(x)}{h}\right)$$

$$= \lim_{h \to 0} f(x+h)\left(\frac{g(x+h) - g(x)}{h}\right) + \lim_{h \to 0} g(x)\left(\frac{f(x+h) - f(x)}{h}\right)$$

$$= f(x)g'(x) + g(x)f'(x)$$