Name: _________________________ Date: ___________________

Learning Goal 3.1	Using all basic derivative rules.

More Questions – Solutions

1. Find the derivative any way your heart desires. Simplify as much as possible.

a. $y = (6x^3 - x)(10 - 2x)$

$$\begin{aligned} y' &= (6x^3 - x)(-2) + (10 - 2x)(18x^2 - 1) \\ &= 2x - 12x^3 + 180x^2 - 10 - 36x^2 + 2x \\ &= -12x^3 + 144x^2 + 4x - 10 \end{aligned}$$

b. $h(x) = \dfrac{4\sqrt{x}}{x^2 - 2}$

$$\begin{aligned} h'(x) &= \frac{(x^2-2)2x^{-1/2} - 4\sqrt{x}(2x)}{(x^2-2)^2} \\ &= \frac{\dfrac{2(x^2-2)}{\sqrt{x}} - 8x\sqrt{x}}{(x^2-2)^2} \\ &= \frac{\dfrac{2(x^2-2) - 8x^2}{\sqrt{x}}}{(x^2-2)^2} \\ &= \frac{2(x^2-2) - 8x^2}{\sqrt{x}(x^2-2)^2} \\ &= \frac{2\sqrt{x}\big((x^2-2) - 4x^2\big)}{x(x^2-2)^2} \\ &= \frac{-2\sqrt{x}(3x^2+2)}{x(x^2-2)^2} \end{aligned}$$

c. $y = \dfrac{4}{x^2}$

$$= 4x^{-2}$$

$$\begin{aligned} y' &= -8x^{-3} \\ &= -\frac{8}{x^3} \end{aligned}$$

d. $f(x) = \dfrac{x^3}{x^3 - 5x + 10}$

$$\begin{aligned} f'(x) &= \frac{(x^2 - 5x + 10)(3x^2) - x^3(3x^2 - 5)}{(x^3 - 5x + 10)^2} \\ &= \frac{3x^4 - 15x^3 + 30x^2 - 3x^5 + 5x^3}{(x^3 - 5x + 10)^2} \\ &= \frac{x^2(-3x^3 + 3x^2 - 10x + 30)}{(x^3 - 5x + 10)^2} \end{aligned}$$

e. $g(x) = \frac{(x-5)^2}{x^{20}}$

$$g(x) = \frac{x^2 - 10x + 25}{x^{20}}$$

$$g'(x) = \frac{x^{20}(2x-10) - 20x^{19}(x^2 - 10x + 25)}{x^{40}}$$

$$= \frac{2x^{21} - 10x^{20} - 20x^{21} + 200x^{20} - 500x^{19}}{x^{40}}$$

$$= \frac{2x^{19}(x^2 - 5x - 10x^2 + 100x - 250)}{x^{40}}$$

$$= \frac{2(-9x^2 + 95x - 250)}{x^{21}}$$

$$= \frac{-2(9x-50)(x-5)}{x^{21}}$$

f. $h(x) = (x^2 + 5x - 3)(x^5)$

$$h'(x) = 5x^4(x^2 + 5x - 3) + x^5(2x+5)$$

$$= 5x^6 + 25x^5 - 15x^4 + 2x^6 + 6x^5$$

$$= 7x^6 + 31x^5 - 15x^4$$

$$= x^4(7x^2 + 31x - 15)$$

g. $y = (x^2 + 5x - 3)(x^{-5})$

$$\frac{dy}{dx} = -5x^{-6}(x^2 + 5x - 3) + x^{-5}(2x+5)$$

$$= -5x^{-4} - 25x^{-5} + 15x^{-6} + 2x^{-4} + 5x^{-5}$$

$$= -x^{-6}(5x^2 + 25x - 15 - 2x^2 - 5x)$$

$$= -\frac{3x^2 + 20x - 15}{x^6}$$

h. $f(x) = (5x^3 + 12x^2 - 15)^{-1}$

$$f(x) = \frac{1}{5x^3 + 12x^2 - 15}$$

$$f'(x) = \frac{(5x^3 + 12x^2 - 15)(0) - (1)(15x^2 + 24x)}{(5x^3 + 12x^2 - 15)^2}$$

$$= \frac{-3x(5x+8)}{(5x^3 + 12x^2 - 15)^2}$$

i. $\frac{d}{dx}(-4x^5 + 3x^3 - 5/x^2)$

$$= \frac{d}{dx}\left(-4x^5 + 3x^3 - \frac{5}{x^2}\right)$$

$$= \frac{d}{dx}(-4x^5 + 3x^3 - 5x^{-2})$$

$$= -20x^4 + 9x^2 + 10x^{-3}$$

$$= -20x^4 + 9x^2 + \frac{10}{x^3}$$

$$= \frac{-20x^7 + 9x^5 + 10}{x^3}$$

Assignment # 23 – 43, 51, 53, 57 – 67, 81, 87 Quiz Next Day!

2. Find an equation for the tangent line at $x = 3$ to

$$f(x) = \frac{x^2 - 4}{5 - x}$$

$$f(3) = \frac{(3)^2 - 4}{5 - (3)} = \frac{9 - 4}{2} = \frac{5}{2}$$

$$f'(x) = \frac{(5 - x)(2x) - (-1)(x^2 - 4)}{(5 - x)^2} = -\frac{x^2 - 10x + 4}{(5 - x)^2}$$

$$f'(3) = -\frac{(3)^2 - 10(3) + 4}{\left(5 - (3)\right)^2}$$

$$f'(3) = -\frac{9 - 30 + 4}{(2)^2}$$

$$f'(3) = \frac{17}{4}$$

$$y - \frac{5}{2} = \frac{17}{4}(x - 3)$$

3. Find a cubic polynomial whose graph has horizontal tangents at $(-2, 5)$ and $(2, 3)$.

$$f(x) = ax^3 + bx^2 + cx + d$$

$$f(-2) = a(-2)^3 + b(-2)^2 + c(-2) + d$$
$$5 = -8a + 4b - 2c + d$$

$$f(2) = a(2)^3 + b(2)^2 + c(2) + d$$
$$3 = 8a + 4b + 2c + d$$

$$f'(x) = 3ax^2 + 2bx + c$$

Horizontal tangents ⇒ local maximum or minimum

$$f'(-2) = f'(2) = 0$$

$$f'(-2) = 3a(-2)^2 + 2b(-2) + c$$
$$0 = 12a - 4b + c$$

$$f'(2) = 3a(2)^2 + 2b(2) + c$$
$$0 = 12a + 4b + c$$

$$\begin{array}{rl} & 5 = -8a + 4b - 2c + d \\ + & 3 = 8a + 4b + 2c + d \\ \hline & 8 = 8b + 2d \\ & d = 4 \end{array}$$

$$\begin{array}{rl} & 0 = 12a - 4b + c \\ - & 0 = 12a + 4b + c \\ \hline & 0 = -8b \\ & b = 0 \end{array}$$

$$\begin{array}{rl} & 5 = -8a + 4b - 2c + d \\ + & 0 = 2(12a + 4b + c) \\ \hline & 5 = 16a + 4 \\ & a = \frac{1}{16} \end{array}$$

$$0 = 12\left(\frac{1}{16}\right) - 4(0) + c$$

$$c = -\frac{3}{4}$$

$$f(x) = \frac{1}{16}x^3 - \frac{3}{4}x + 4$$