Name: _____ Date: _____

Learning Goal 3.2	Factoring, including the factor theorem and the remainder theorem.

Example Divide each of the following and provide a division statement. Identify any restrictions on the variable.

a. $x^3 - 4x^2 + 5x - 1$ by $x - 5$ (long division)

$x = 5$

$$x-5 \;\big|\; x^3 - 4x^2 + 5x - 1$$

$x^2 + x + 10$

$-(x^3 - 5x^2)$

$x^2 + 5x$

$-(x^2 - 5x)$

$10x - 1$

$-(10x - 50)$

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b. $x^3 - 4x^2 + 5x - 1$ by $x + 1$ (synthetic division)

$x = -1$

-1	1	-4	5	-1
	$\downarrow$	-1	5	-10
	1	-5	10	-11

$x^3 - 4x^2 + 5x - 1 = (x+1)(x^2 - 5x + 10) - 11$

The Remainder Theorem

When a polynomial in x, $P(x)$ is divided by $x-a$, the remainder is $P(a)$

Check: $(5)^3 - 4(5)^2 + 5(5) - 1$

$= 125 - 100 + 25 - 1$

$= 49$

$(-1)^3 - 4(-1)^2 + 5(-1) - 1$

$= -1 - 4 - 5 - 1$

$= -11$

Example Find the remainder of the quotient

$$\frac{x^4 - x^3 + 3x - 5}{x + 2} \rightarrow f(x)$$

a. using long division

$x^3 - 3x^2 + 6x - 9$

$$x+2 \;\big|\; x^4 - x^3 + 0x^2 + 3x - 5$$

$-(x^4 + 2x^3)$

$-3x^3 + 0x^2$

$-(-3x^3 - 6x^2)$

$6x^2 + 3x$

$-(6x^2 + 12x)$

$-9x - 5$

$-(-9x - 18)$

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b. using synthetic division

-2	1	-1	0	3	-5
	$\downarrow$	-2	6	-12	18
	1	-3	6	-9	13

c. using the remainder theorem

$f(-2)$

$= (-2)^4 - (-2)^3 + 3(-2) - 5$

$= 16 + 8 - 6 - 5$

$= 13$

Assignment p. 124 # 1 – 16 Quiz Next Day!

$f(x) =$

Example When $x^3 + 3x^2 - kx + 10$ is divided by $x - 5$, the remainder is 15. Find the value of k.

$x = 5$

$f(5) = 15 = (5)^3 + 3(5)^2 - k(5) + 10$

$15 = 125 + 75 - 5k + 10$

$15 = 210 - 5k$

$-210 \quad -210$

$-195 = -5k$

$\Rightarrow \quad k = 39$

Example When $P(x) = 3x^3 + mx^2 + nx - 7$ is divided by $x - 2$, the remainder is -3. When $P(x)$ is divided by $x + 1$, the remainder is -18. What are the values of m and n.

$x = 2$ $\qquad$ $x = -1$

$P(2) = -3 = 3(2)^3 + m(2)^2 + n(2) - 7$

$-3 = 24 + 4m + 2n - 7$

$-20 = 4m + 2n$

$-20 = 4(n - 8) + 2n$

$-20 = 4n - 32 + 2n$

$\frac{12}{6} = \frac{6n}{6}$

$n = 2$

$P(-1) = -18 = 3(-1)^3 + m(-1)^2 + n(-1) - 7$

$-18 = -3 + m - n - 7$

$-8 = m - n$

$+n \qquad +n$

$m = n - 8$

$= 2 - 8$

$m = -6$

Exmaple $f(x) = x^4 + 3x^2 + 2$ has the same remainder as $g(x) = x^5 + mx^4 + x^3$ when divided by $x - 2$. Determine the value of m.

$x = 2$

$f(2) = (2)^4 + 3(2)^2 + 2$

$= 16 + 12 + 2$

$= 30$

$g(2) = (2)^5 + m(2)^4 + (2)^3$

$= 32 + 16m + 8$

$= 40 + 16m$

$30 = 40 + 16m$

$-10 = 16m$

$m = -\frac{10}{16} = -\frac{5}{8}$