Name: _____ Date: _____

Learning Goal 3.3	Using more derivative rules.

More Questions – Solutions

1. Use implicit differentiation to find the following derivatives.

a. $y^2 = 1 + x^2$

$$2y\frac{dy}{dx} = 2x$$
$$\frac{dy}{dx} = \frac{2x}{2y}$$
$$= \frac{x}{y}$$

b. $x^2 + xy + y^2 = 7$

$$2x + \left(x\frac{dy}{dx} + y(1)\right) + 2y\frac{dy}{dx} = 0$$
$$2x + \frac{dy}{dx}(x + 2y) + y = 0$$
$$\frac{dy}{dx}(x + 2y) = -(y + 2x)$$
$$\frac{dy}{dx} = -\frac{y + 2x}{x + 2y}$$

c. $x^3 + xy^2 = y^3 + yx^2$

$$3x^2 + \left(x\left(2y\frac{dy}{dx}\right) + y^2(1)\right) = 3y^2\frac{dy}{dx} + \left(y(2x^2) + x^2\left(\frac{dy}{dx}\right)\right)$$
$$3x^2 + 2xy\frac{dy}{dx} + y^2 = 3y^2\frac{dy}{dx} + 2x^2y + x^2\frac{dy}{dx}$$
$$\frac{dy}{dx}(2xy - 3y^2 - x^2) = 2x^2y - 3x^2 - y^2$$
$$\frac{dy}{dx} = \frac{2x^2y - 3x^2 - y^2}{2xy - 3y^2 - x^2}$$

d. $4\cos x \sin y = 1$

$$4\cos x\cos y\frac{dy}{dx} - 4\sin x\sin y = 0$$
$$4\cos x\cos y\frac{dy}{dx} = 4\sin x\sin y$$
$$\frac{dy}{dx} = \frac{4\sin x\sin y}{4\cos x\cos y}$$
$$\frac{dy}{dx} = \frac{\sin x\sin y}{\cos x\cos y}$$
$$\frac{dy}{dx} = \tan x\tan y$$

e. $\sqrt{x}+\sqrt{y}=9$

$$x^{1/2}+y^{1/2}=9$$

$$\frac{1}{2}x^{-1/2}+\frac{1}{2}y^{-1/2}\frac{dy}{dx}=0$$

$$\frac{\sqrt{x}}{2x}+\frac{\sqrt{y}}{2y}\frac{dy}{dx}=0$$

$$\frac{\sqrt{y}}{2y}\frac{dy}{dx}=-\frac{\sqrt{x}}{2x}$$

$$\frac{dy}{dx}=-\frac{\sqrt{x}}{2x}\times\frac{2y}{\sqrt{y}}$$

$$\frac{dy}{dx}=-\frac{y\sqrt{x}}{x\sqrt{y}}$$

$$\frac{dy}{dx}=-\frac{\sqrt{xy}}{x}$$

f. $\tan\left(\frac{x}{y}\right)=x+y$

$$\sec^2\left(\frac{x}{y}\right)\times\left(\frac{y-x\frac{dy}{dx}}{y^2}\right)=1+\frac{dy}{dx}$$

$$\sec^2\left(\frac{x}{y}\right)\times\left(y-x\frac{dy}{dx}\right)=y^2\left(1+\frac{dy}{dx}\right)$$

$$y\sec^2\left(\frac{x}{y}\right)-x\frac{dy}{dx}\sec^2\left(\frac{x}{y}\right)=y^2+y^2\frac{dy}{dx}$$

$$y\sec^2\left(\frac{x}{y}\right)+y^2=y^2\frac{dy}{dx}+x\frac{dy}{dx}\sec^2\left(\frac{x}{y}\right)$$

$$y\sec^2\left(\frac{x}{y}\right)+y^2=\frac{dy}{dx}\left(y^2+x\sec^2\left(\frac{x}{y}\right)\right)$$

$$\frac{dy}{dx}=\frac{y\sec^2\left(\frac{x}{y}\right)+y^2}{y^2+x\sec^2\left(\frac{x}{y}\right)}$$

g. $\sin(x+y)=xy$

$$\cos(x+y)\times\left(1+\frac{dy}{dx}\right)=y+x\frac{dy}{dx}$$

$$\cos(x+y)+\cos(x+y)\frac{dy}{dx}=y+x\frac{dy}{dx}$$

$$\cos(x+y)\frac{dy}{dx}-x\frac{dy}{dx}=y-\cos(x+y)$$

$$\frac{dy}{dx}(\cos(x+y)-x)=y-\cos(x+y)$$

$$\frac{dy}{dx}=\frac{y-\cos(x+y)}{\cos(x+y)-x}$$

h. $\frac{1}{x}+\frac{1}{y}=7$

$$x^{-1}+y^{-1}=7$$

$$-x^{-2}-y^{-2}\frac{dy}{dx}=0$$

$$-y^{-2}\frac{dy}{dx}=x^{-2}$$

$$-\frac{1}{y^2}\frac{dy}{dx}=\frac{1}{x^2}$$

$$\frac{dy}{dx}=-\frac{y^2}{x^2}$$

i. $y = (x+1)^2(x+2)^3$

$\ln y = \ln((x+1)^2(x+2)^3)$

$\ln y = \ln((x+1)^2) + \ln((x+2)^3)$

$\ln y = 2\ln(x+1) + 3\ln(x+2)$

$\frac{1}{y}\frac{dy}{dx} = 2\left(\frac{1}{x+1}\right) + 3\left(\frac{1}{x+2}\right)$

$\frac{1}{y}\frac{dy}{dx} = \frac{2(x+2)+3(x+1)}{(x+1)(x+2)}$

$\frac{dy}{dx} = \frac{2(x+2)+3(x+1)}{(x+1)(x+2)} \times y$

$\frac{dy}{dx} = \frac{2(x+2)+3(x+1)}{(x+1)(x+2)} \times (x+1)^2(x+2)^3$

$\frac{dy}{dx} = \big(2(x+2)+3(x+1)\big) \times (x+1)(x+2)^2$

$\frac{dy}{dx} = (2x+4+3x+3)(x+1)(x+2)^2$

$\frac{dy}{dx} = (5x+7)(x+1)(x+2)^2$

j. $y = (3x+2)^4(5x-1)^2$

$\ln y = \ln((3x+2)^4(5x-1)^2)$

$\ln y = \ln((3x+2)^4) + \ln((5x-1)^2)$

$\ln y = 4\ln(3x+2) + 2\ln(5x-1)$

$\frac{1}{y}\frac{dy}{dx} = 4\left(\frac{1}{3x+2} \times 3\right) + 2\left(\frac{1}{5x-1} \times 5\right)$

$\frac{1}{y}\frac{dy}{dx} = \frac{12}{3x+2} + \frac{10}{5x-1}$

$\frac{1}{y}\frac{dy}{dx} = \frac{12(5x-1)+10(3x+2)}{(3x+2)(5x-1)}$

$\frac{dy}{dx} = \frac{12(5x-1)+10(3x+2)}{(3x+2)(5x-1)} \times y$

$\frac{dy}{dx} = \frac{12(5x-1)+10(3x+2)}{(3x+2)(5x-1)} \times (3x+2)^4(5x-1)^2$

$\frac{dy}{dx} = \big(12(5x-1)+10(3x+2)\big) \times (3x+2)^3(5x-1)$

$\frac{dy}{dx} = (60x-12+30x+20) \times (3x+2)^3(5x-1)$

$\frac{dy}{dx} = (90x-8)(3x+2)^3(5x-1)$

$\frac{dy}{dx} = 2(45x-4)(3x+2)^3(5x-1)$

k. $y = (x-1)^2(x+1)^3(x+3)^4$

$$\ln y = \ln((x-1)^2(x+1)^3(x+3)^4)$$
$$\ln y = \ln((x-1)^2) + \ln((x+1)^3) + \ln((x+3)^4)$$
$$\ln y = 2\ln(x-1) + 3\ln(x+1) + 4\ln(x+3)$$

$$\frac{1}{y}\frac{dy}{dx} = 2\left(\frac{1}{x-1}\right) + 3\left(\frac{1}{x+1}\right) + 4\left(\frac{1}{x+3}\right)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x-1} + \frac{3}{x+1} + \frac{4}{x+3}$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{2(x+1)(x+3) + 3(x-1)(x+3) + 4(x-1)(x+1)}{(x-1)(x+1)(x+3)}$$
$$\frac{dy}{dx} = \frac{2(x+1)(x+3) + 3(x-1)(x+3) + 4(x-1)(x+1)}{(x-1)(x+1)(x+3)} \times y$$

$$\frac{dy}{dx} = \big(2(x+1)(x+3) + 3(x-1)(x+3) + 4(x-1)(x+1)\big) \times (x-1)(x+1)^2(x+3)^3$$
$$\frac{dy}{dx} = \big(2(x^2+4x+3) + 3(x^2+2x-3) + 4(x^2-1)\big) \times (x-1)(x+1)^2(x+3)^3$$
$$\frac{dy}{dx} = (2x^2+8x+6+3x^2+6x-9+4x^2-4) \times (x-1)(x+1)^2(x+3)^3$$
$$\frac{dy}{dx} = (9x^2+14x-7)(x-1)(x+1)^2(x+3)^3$$

I. $y = \dfrac{\sqrt{4+3x^2}}{\sqrt[3]{x^2+1}}$

$$y = (4+3x^2)^{1/2}(x^2+1)^{-1/3}$$
$$\ln y = \ln\left((4+3x^2)^{1/2}(x^2+1)^{-1/3}\right)$$
$$\ln y = \ln\left((4+3x^2)^{1/2}\right) + \ln\left((x^2+1)^{-1/3}\right)$$
$$\ln y = \frac{1}{2}\ln(4+3x^2) - \frac{1}{3}\ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{4+3x^2}\times 6x\right) - \frac{1}{3}\left(\frac{1}{x^2+1}\times 2x\right)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{3x}{4+3x^2} - \frac{2x}{3(x^2+1)}$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{9x(x^2+1) - 2x(4+3x^2)}{(4+3x^2)(x^2+1)}$$
$$\frac{dy}{dx} = \frac{9x(x^2+1) - 2x(4+3x^2)}{(4+3x^2)(x^2+1)}\times y$$

$$\frac{dy}{dx} = \frac{9x(x^2+1) - 2x(4+3x^2)}{(4+3x^2)(x^2+1)}\times (4+3x^2)^{1/2}(x^2+1)^{-1/3}$$
$$\frac{dy}{dx} = \frac{9x(x^2+1) - 2x(4+3x^2)}{(4+3x^2)^{1/2}(x^2+1)^{4/3}}$$
$$\frac{dy}{dx} = \frac{9x^3+9x-8x-6x^3}{(4+3x^2)^{1/2}(x^2+1)^{4/3}}$$
$$\frac{dy}{dx} = \frac{3x^3+x}{(4+3x^2)^{1/2}(x^2+1)^{4/3}}$$
$$\frac{dy}{dx} = \frac{x(3x^2+1)}{(4+3x^2)^{1/2}(x^2+1)^{4/3}}$$
$$\frac{dy}{dx} = \frac{x(3x^2+1)(4+3x^2)^{1/2}(x^2+1)^{2/3}}{(4+3x^2)(x^2+1)^2}$$
$$\frac{dy}{dx} = \frac{x(3x^2+1)(4+3x^2)^{3/6}(x^2+1)^{4/6}}{(4+3x^2)(x^2+1)^2}$$
$$\frac{dy}{dx} = \frac{x(3x^2+1)\sqrt[6]{(4+3x^2)^3(x^2+1)^4}}{(4+3x^2)(x^2+1)^2}$$

Play Day