

Name: _____

Date: _____

Learning Goal 4.2

Given a quadratic equation, find the values of solution(s) by factoring, completing the square or using the quadratic formula.

When factoring fails you ... there are 2 other methods.

Example Solve the following equations by completing the square.

a. $x^2 - 4x = 11$

$-11 \quad -11$

$x^2 - 4x - 11 = 0$

$(x^2 - 4x + 4 - 4) - 11 = 0$
 $(x^2 - 4x + 4) - 4 - 11 = 0$

$(x-2)^2 - 15 = 0$
 $+15 \quad +15$

$\sqrt{(x-2)^2} = \sqrt{15}$

$x-2 = \pm \sqrt{15}$
 $+2 \quad +2$

$x = 2 \pm \sqrt{15}$

c. $2x^2 + 7x = -6$
 $+6 \quad +6$

$2x^2 + 7x + 6 = 0 \leftarrow \text{is factorable}$

$2(x^2 + \frac{7}{2}x) + 6 = 0 \quad (\frac{7}{4})^2 = \frac{49}{16}$

$2(x^2 + \frac{7}{2}x + \frac{49}{16} - \frac{49}{16}) + 6 = 0$

$2(x^2 + \frac{7}{2}x + \frac{49}{16}) - \frac{49}{8} + \frac{48}{8} = 0$

$2(x + \frac{7}{4})^2 - \frac{1}{8} = 0 \Rightarrow \checkmark$

$\frac{1}{8} \quad +\frac{1}{8}$
 $\frac{2(x + \frac{7}{4})^2}{2} = \frac{1}{8} \div 2$
 $\sqrt{\frac{(x + \frac{7}{4})^2}{4}} = \sqrt{\frac{1}{16}}$

$x + \frac{7}{4} = \pm \frac{1}{4}$
 $x = -\frac{7}{4} \pm \frac{1}{4}$

b. $x^2 - 21 = -10x$

$+10x \quad +10x$

$(\frac{10}{2})^2 = (5)^2$
 $= 25$

$x^2 + 10x - 21 = 0$

$(x^2 + 10x + 25 - 25) - 21 = 0$

$(x^2 + 10x + 25) - 25 - 21 = 0$

$(x+5)^2 - 46 = 0 \Rightarrow \checkmark$

$\sqrt{(x+5)^2} = \sqrt{46}$

$x+5 = \pm \sqrt{46}$
 $-5 \quad -5$

$x = -5 \pm \sqrt{46}$

d. $2x^2 + 3x = 7$

$-7 \quad -7$

$2x^2 + 3x - 7 = 0$

$2(x^2 + \frac{3}{2}x) - 7 = 0$

$(\frac{3}{4})^2 = \frac{9}{16}$

$2(x^2 + \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}) - 7 = 0$

$2(x^2 + \frac{3}{2}x + \frac{9}{16}) - \frac{9}{8} - \frac{56}{8} = 0$

$2(x + \frac{3}{4})^2 - \frac{65}{8} = 0 \Rightarrow \checkmark$

$2(x + \frac{3}{4})^2 = \frac{65}{8}$

$(x + \frac{3}{4})^2 = \frac{65}{16}$

$x + \frac{3}{4} = \pm \sqrt{\frac{65}{16}}$

$x = -\frac{3}{4} \pm \frac{\sqrt{65}}{4}$

B
E
D
M
A
S

$$x = -\frac{b}{2a} = -\frac{3}{2}$$

Example The circular Toonie coin consists of an aluminum and bronze core and a nickel outer ring. If the radius of the inner core is 0.84 cm and the area of the circular face of the coin is $1.96\pi \text{ cm}^2$, what is the width of the outer ring?

↳ diameter of the coin?



$$A = 1.96\pi$$

$$\begin{aligned} d &= 2r \\ &= 2\left(\frac{7}{5}\right) \\ &= \frac{14}{5} \text{ cm} \\ &= 2.8 \text{ cm} \end{aligned}$$

$$2.8 - 0.84 = 1.96$$

The width of the outer ring is 1.96 cm.

$$\begin{aligned} A &= \pi r^2 = \frac{1.96\pi}{\pi} \\ \sqrt{r^2} &= \sqrt{1.96} \\ r &= \pm \sqrt{1.96} \\ &= \pm \sqrt{\frac{196}{100}} \\ &= \pm \frac{14}{10} \\ &= \pm \frac{7}{5} \end{aligned}$$

Example A defender kicks a soccer ball away from her own goal. The path of the kicked soccer ball can be approximated by the quadratic function

$$h(x) = -0.06x^2 + 3.168x - 35.34$$

where x is the horizontal distance travelled in metres, from the goal line and h is the height.

a. How far is the soccer ball from the goal line when it is kicked?

$$\begin{aligned} h(x) &= -0.06(x^2 - 52.8x) - 35.34 \\ &= -0.06(x^2 - 52.8x + 696.96 - 696.96) - 35.34 \\ &= -0.06(x^2 - 52.8x + 696.96) + 41.8176 - 35.34 \\ &= -0.06(x - 26.4)^2 + 6.4776 = 0 \\ -0.06(x - 26.4)^2 &= -6.4776 \\ (x - 26.4)^2 &= 107.96 \end{aligned}$$

$$\left(\frac{52.8}{2}\right)^2 = 696.96$$

b. How far does the soccer ball travel before it hits the ground?

$$x - 26.4 = \pm 10.39$$

$$x = 26.4 \pm 10.39$$

$$x_1 = 16.01$$

$$x_2 = 36.79$$

a. The ball is ~16 m from the goal line when it is kicked

b. The ball travels ~20.8 m before it hits the ground.