Name: ________________________ Date: __________________

Learning Goal 4.1	The Mean Value Theorem and L'Hospital's Rule

More Questions – Solutions

1. Suppose that we know $f(x)$ is continuous and differentiable on $[6, 15]$. Let's also suppose that we know $f(6) = -2$ and $f'(x) \leq 10$. What is the largest possible value for $f(15)$?

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$
$$f'(c) = \frac{f(15) - f(6)}{15 - 6}$$
$$f'(c) = \frac{f(15) + 2}{9}$$
$$9f'(c) = f(15) + 2$$

$$f(15) = 9f'(c) - 2$$
$$f(15) \leq 9(10) - 2$$
$$f(15) \leq 88$$

2. A car travels 180 km in 2 hours. Its speedometer must have read how fast at least once?

$$\begin{aligned} \text{average speed} &= \text{secant slope} \\ &= \frac{180 - 0}{2 - 0} \\ &= 90 \end{aligned}$$

Since the instantaneous speed, or that read by the speedometer, is the same as the tangent slope, the by the MVT $f'(x) = 90\ {}^{\text{km}}/_{\text{h}}$ at least once.

3. Suppose that f is a differentiable function such that $f'(x) \leq 2$ for all x. What is the largest possible value of $f(7)$ if $f(3) = 5$?

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$
$$f'(c) = \frac{f(7) - f(3)}{7 - 3}$$
$$f'(c) = \frac{f(7) - 5}{4}$$
$$4f'(c) = f(7) - 5$$

$$f(7) = 4f'(c) + 5$$
$$f(7) \leq 4(2) + 5$$
$$f(7) \leq 13$$

4. Let $f(x) = x^2$. Find a value $c \in (-1, 2)$ so that $f'(c)$ equals the slope between the endpoints of $f(x)$ on $[-1, 2]$.

$$f(-1) = (-1)^2 = 1 \qquad f(2) = (2)^2 = 4$$

$$\text{secant slope} = \frac{f(2) - f(-1)}{2 - (-1)} = \frac{4 - 1}{3} = 1$$

$$f'(x) = 2x$$
$$f'(c) = 1$$
$$2x = 1$$
$$x = \frac{1}{2}$$