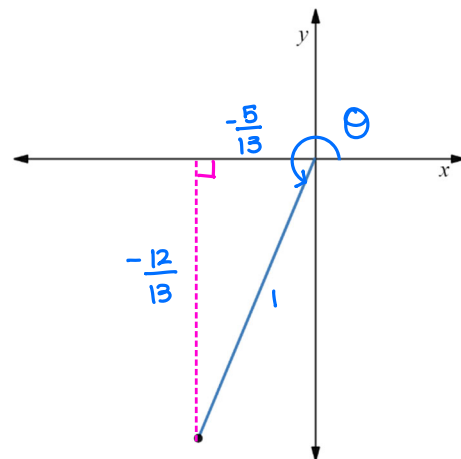Name: _____   Date: _____

Learning Goal 4.1	Examining angles in standard position in both radians and degrees. Exploring the unit circle, reference and coterminal angles and special angles.

More Questions – Solutions

1. The point $\left(-5/13, -12/13\right)$ is the point of intersection of the terminal arm of the unit circle and angle, θ, in standard position.
   a. Draw θ.
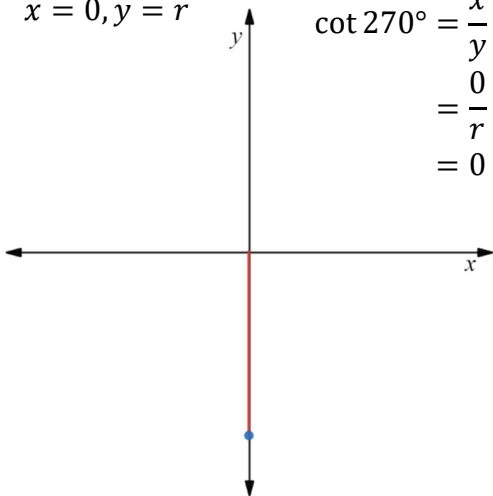   b. Find the exact value of the six trigonometric ratios for θ.

$$\sin\theta = -\frac{12}{13} \qquad \cos\theta = -\frac{5}{13} \qquad \tan\theta = \frac{12}{5}$$
$$\csc\theta = -\frac{13}{12} \qquad \sec\theta = -\frac{13}{5} \qquad \cot\theta = \frac{5}{12}$$

2. Find the exact value of each of the following, include a sketch.

   a. $\cot 270°$

   $x = 0, y = r$

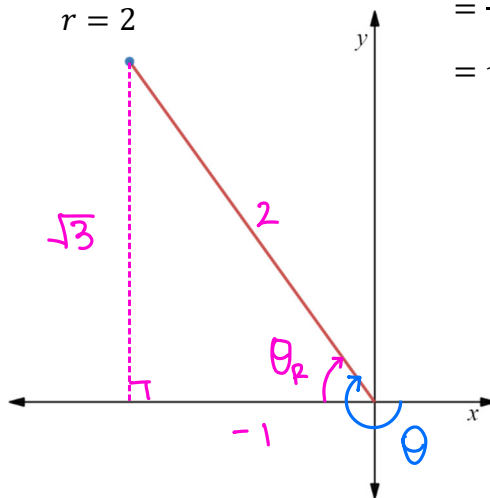 $$\cot 270° = \frac{x}{y}$$
   $$= \frac{0}{r}$$
   $$= 0$$

   b. $\tan\left(-\frac{4\pi}{3}\right)$

   $$\theta_R = \frac{\pi}{3}$$
   $$x = 1, y = \sqrt{3}$$
   $$r = 2$$

   $$\tan\left(-\frac{4\pi}{3}\right) = \frac{y}{x}$$
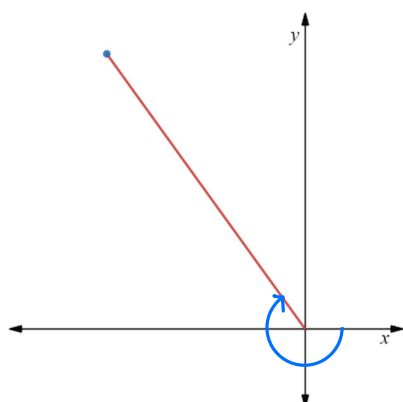 $$= \frac{\sqrt{3}}{1}$$
   $$= \sqrt{3}$$

3. Find the approximate value of each of the following. Include a sketch. Round your answer to three decimal places.
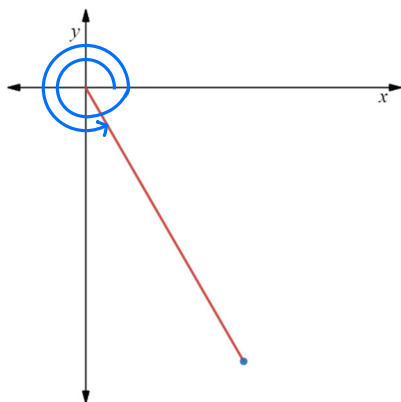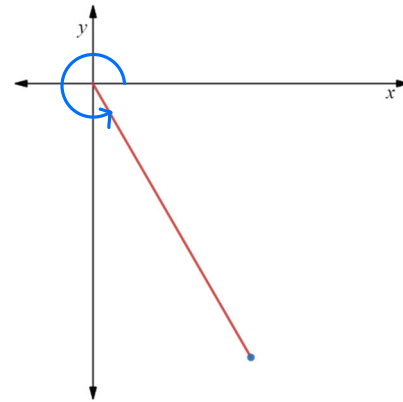
   a. $\cos(678°) \approx 0.743$ b. $\csc(-3.98) \approx 1.345$ c. $\cot\left(\dfrac{9\pi}{5}\right) \approx -1.376$

4. Solve each of the following equations, $0 \le \theta < 2\pi$.

   a. $\cos\theta = -0.366$

$$\theta_1 = \cos^{-1}(-0.366)$$
$$= 1.946$$

$$\theta_2 = \pi + \theta_1$$
$$= \pi + 1.196$$
$$= 4.337$$

$$\theta_R = \pi - \theta_1$$
$$= \pi - 1.946$$
$$= 1.196$$

   b. $\csc\theta = 1.678$

$$\sin\theta = \frac{1}{1.678}$$
$$= 0.596$$

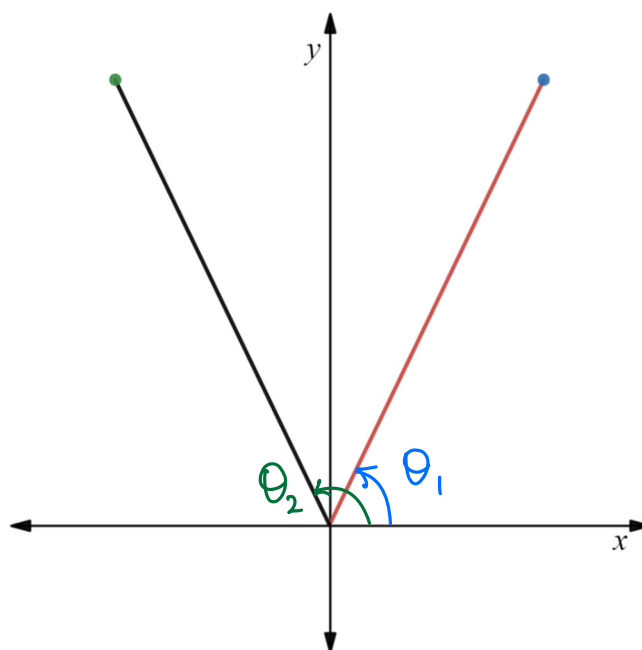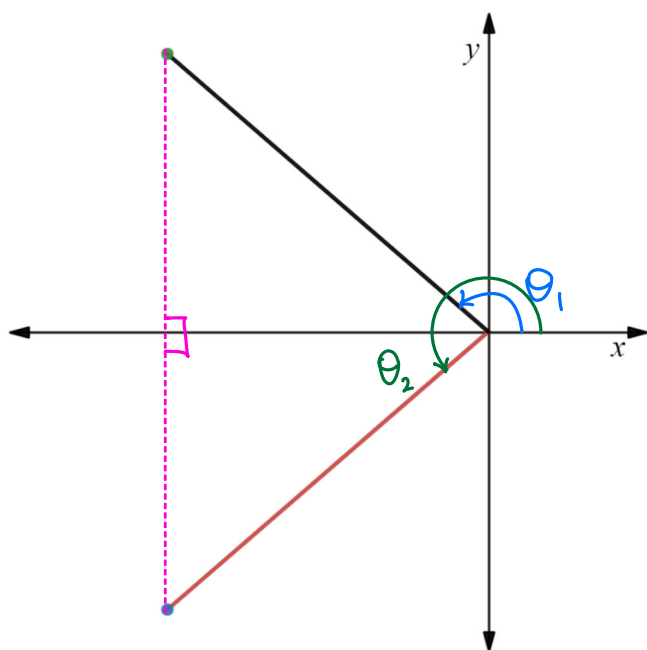$$\theta_1 = \sin^{-1}(0.596)$$
$$= 0.638$$

$$\theta_R = \theta_1$$
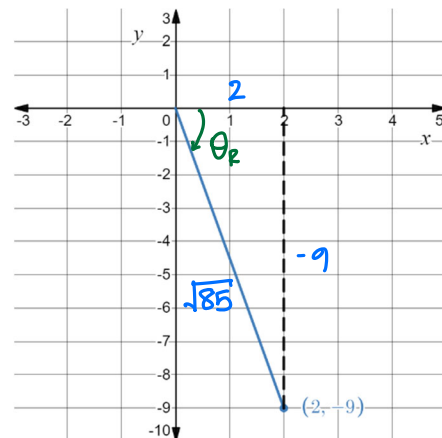
$$\theta_2 = \pi - \theta_1$$
$$= \pi - 0.638$$
$$= 2.503$$

5. The point $(2, -9)$ is on the terminal arm of an angle, θ, in standard position.
    a. Draw θ.
    b. Find the exact value of the six trigonometric ratios for θ.

$$r^2 = x^2 + y^2$$
$$= (2)^2 + (-9)^2$$
$$= 4 + 81$$
$$= 85$$
$$r = \sqrt{85}$$

$$\sin \theta = -\frac{9}{\sqrt{85}} \qquad \cos \theta = \frac{2}{\sqrt{85}} \qquad \tan \theta = -\frac{9}{2}$$
$$= -\frac{9\sqrt{85}}{85} \qquad = \frac{2\sqrt{85}}{85}$$
$$\csc \theta = -\frac{\sqrt{85}}{9} \qquad \sec \theta = \frac{\sqrt{85}}{2} \qquad \cot \theta = -\frac{2}{9}$$
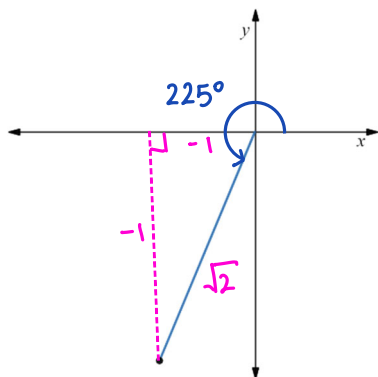
6. Find the exact value of each of the following, where possible. Include a sketch.

    a. $\tan(225°)$

    $$\theta_R = 225 - 180$$
    $$= 45°$$

    $$\tan 225° = \frac{-1}{-1} = 1$$

    b. $\sin(540°)$

    $$\theta_R = 540 - 360$$
    $$= 180°$$
    $$= 0°$$

    $$\sin(540°) = \sin(180°)$$
    $$= 0$$

    c. $\cos\left(\frac{5\pi}{6}\right)$

    $$\theta_R = \pi - \frac{5\pi}{6}$$
    $$= \frac{\pi}{6}$$

    $$\cos\left(\frac{5\pi}{6}\right) = \frac{-\sqrt{3}}{2}$$

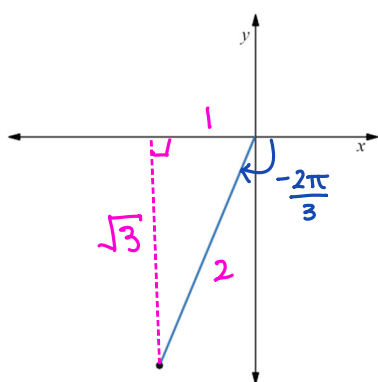d. $\csc\left(-\dfrac{2\pi}{3}\right)$

e. $\cot(-560°)$

f. $\sec\left(\dfrac{\pi}{5}\right)$

$$\csc\left(-\frac{2\pi}{3}\right) = \csc\left(\frac{4\pi}{3}\right)$$
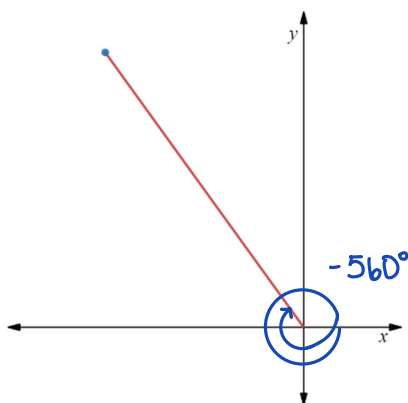
$$\theta_R = \frac{4\pi}{3} - \frac{3\pi}{3}$$

$$= \frac{\pi}{3}$$

$$\csc\left(\frac{4\pi}{3}\right) = \frac{1}{\sin\left(\frac{4\pi}{3}\right)}$$

$$= \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)}$$

$$= -\frac{2}{\sqrt{3}}$$

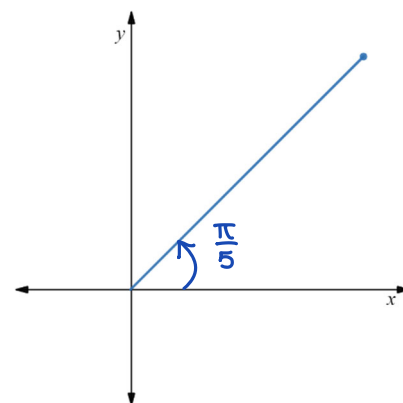$$\cot(-560°) = \cot(-200°)$$

$$= \cot(160°)$$

$$= \frac{1}{\tan(160°)}$$

$$= -2.747$$

$$\sec\left(\frac{\pi}{5}\right) = \frac{1}{\cos\left(\frac{\pi}{5}\right)}$$

$$= 1.236$$

7.   Solve each of the following equations, $0 \le \theta < 2\pi$. Find exact values where possible, otherwise round to the nearest thousandth of a radian.

a. $\cot\theta = -\sqrt{3}$

$$\tan\theta = -\frac{1}{\sqrt{3}}$$

$$\tan\theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

$$\theta \approx -0.651$$

$$\theta_R \approx 0.651$$

$$\theta_1 = 2\pi - \theta_R \qquad \theta_2 = \pi - \theta_R$$

$$= 2\pi - 0.651 \qquad\quad = \pi - 0.651$$

$$\approx 5.632 \qquad\qquad\quad \approx 2.491$$

b. $\sin\theta = -\dfrac{1}{2}$

$$\theta_R = \frac{\pi}{6}$$

$$\theta_1 = \pi + \theta_R \qquad \theta_2 = 2\pi - \theta_R$$

$$= \pi + \frac{\pi}{6} \qquad\qquad = 2\pi - \frac{\pi}{6}$$

$$= \frac{7\pi}{6} \qquad\qquad\quad = \frac{11\pi}{6}$$

8. Solve each of the following equations, $-360° \leq \theta < 360°$. Find exact values where possible, otherwise round to the nearest tenth of a degree.

    a. $\cos \theta = -0.366$

$$\theta \approx 111.5°$$
$$\theta_R \approx 180 - \theta$$
$$\approx 68.5°$$

$\theta_1 \approx 111.5°$ $\theta_2 = 180 + \theta_R$
$$\approx 248.5°$$

$\theta_3 = -180 + \theta_R$ $\theta_4 = -180 - \theta_R$
$$\approx -111.5° \qquad\qquad \approx -248.5°$$

    b. $\csc \theta = \sqrt{2}$

$$\sin \theta = \frac{1}{\sqrt{2}}$$
$$\theta_R = 45°$$

$\theta_1 = \theta_R$ $\theta_2 = 180 - \theta_R$
$$= 45° \qquad\qquad\quad = 135°$$

$\theta_3 = -180 - \theta_R$ $\theta_4 = -360 + \theta_R$
$$= -225 \qquad\qquad\quad = -315$$

9. Solve each of the following equations, $-\pi \leq \theta < 2\pi$. Find exact values where possible, otherwise round to the nearest thousandth of a radian.

    a. $3 \cos \theta = \cos \theta + 1$

$$2 \cos \theta = 1$$
$$\cos \theta = \frac{1}{2}$$
$$\theta_R = \frac{\pi}{3}$$

$\theta_1 = \frac{\pi}{3}$ $\theta_2 = 2\pi - \theta_R$
$$= \frac{5\pi}{3}$$

$\theta_3 = 0 - \theta_R$
$$= -\frac{\pi}{3}$$

    b. $\sqrt{3} \cot \theta = 1$

$$\cot \theta = \frac{1}{\sqrt{3}}$$
$$\tan \theta = \sqrt{3}$$
$$\theta_R = \frac{\pi}{3}$$

$\theta_1 = \frac{\pi}{3}$ $\theta_2 = \pi + \theta_R$
$$= \frac{4\pi}{3}$$

$\theta_3 = -\pi + \theta_R$
$$= -\frac{2\pi}{3}$$