Name: ___________________________ Date: ____________________

Learning Goal 6.1	Using identities to reduce complexity in expressions and solve equations.

Double Angle Identities

$\sin^2 A = 1 - \cos^2 A$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

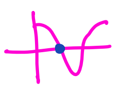
$\cos^2 A = 1 - \sin^2 A$

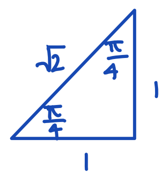
$$\sin 2A = 2\sin A \cos A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Example Verify the identity numerically.

$$\cos 2A = \cos^2 A - \sin^2 A, \quad \measuredangle A = \frac{\pi}{4}$$

LS	RS
$= \cos\left(2 \times \frac{\pi}{4}\right)$	$= \cos^2\left(\frac{\pi}{4}\right) - \sin^2\left(\frac{\pi}{4}\right)$
$= \cos\left(\frac{\pi}{2}\right)$	$= \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$
$= 0$	$= \frac{1}{2} - \frac{1}{2}$
	$=$





Example Express the following as a single trigonometric function of a single angle (do not evaluate). * Matching Questions

a. $2\sin\left(\frac{\pi}{5}\right)\cos\left(\frac{\pi}{5}\right)$

$\sin(2A) = 2\sin A\cos A$

$A = \frac{\pi}{5}$

$2A = \frac{2\pi}{5}$

$2\sin\left(\frac{\pi}{5}\right)\cos\left(\frac{\pi}{5}\right) = \sin\left(\frac{2\pi}{5}\right)$

b. $4\cos^2 35° - 2$

$= 4\cos^2\theta - 2$

$= 2(2\cos^2\theta - 1)$

$= 2(\cos 2\theta)$

$= 2\cos(70°)$

c. $\sin 60° \cos 60°$

$\frac{\sin 2\theta}{2} = \frac{2\sin\theta\cos\theta}{2}$

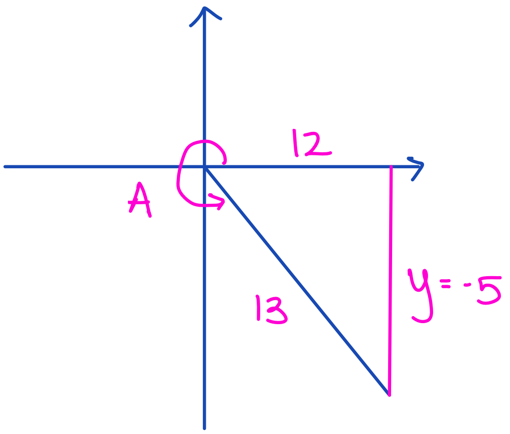
$= \frac{\sin(2 \times 60)}{2}$

$= \frac{\sin(120)}{2}$

* we can evaluate without a calc, but the question told us not to.

Assignment p. 306 # 1(ce), 2(c), 3 – 5, 8(bdf), 11 – 18, 21, 24, C1 Quiz Next Day!

Example If $\cos A = {}^{12}/_{13}$ and $\measuredangle A$ is in the fourth quadrant, find the exact value of $\sin 2A$.



$x^2 + y^2 = r^2$
$(12)^2 + y^2 = (13)^2$
$144 + y^2 = 169$
$y^2 = 25$
$y = \pm\sqrt{25}$
$= \pm 5$

$\sin 2A = 2\sin A\cos A$
$= 2\left(-\frac{5}{13}\right)\left(\frac{12}{13}\right)$
$= -\frac{120}{169}$

Example Prove the following equation.

$$\frac{1+\cos 2x}{\sin 2x} = \cot x$$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Left side:

$= \frac{1 + (2\cos^2 x - 1)}{2\sin x \cos x}$

$= \frac{2\cos^2 x}{2\sin x\cos x}$

$= \frac{\cos^2 x}{\sin x \cos x}$

$= \frac{\cos x}{\sin x}$

Right side:

$= \frac{\cos x}{\sin x}$