Name: ________________________ Date: __________________

Learning Goal 6.1	Using identities to reduce complexity in expressions and solve equations.

More Questions – Solutions

Double Angle Identities

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

$$\sin 2A = 2\sin A \cos A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

1. Prove the identities using the Pythagorean identities.

a. $\cos 2A = 2\cos^2 A - 1$

$$\sin^2 A + \cos^2 A = 1$$
$$\sin^2 A = 1 - \cos^2 A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= \cos^2 A - (1 - \cos^2 A)$$
$$= 2\cos^2 A - 1$$

b. $\cos 2A = 1 - 2\sin^2 A$

$$\sin^2 A + \cos^2 A = 1$$
$$\cos^2 A = 1 - \sin^2 A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= (1 - \sin^2 A) - \sin^2 A$$
$$= 1 - 2\sin^2 A$$

2. Express the following as a single trigonometric function of a single angle (do not evaluate).

a. $2\sin\left(\frac{\pi}{5}\right)\cos\left(\frac{\pi}{5}\right)$

$$= \sin 2\left(\frac{\pi}{5}\right)$$
$$= \sin\left(\frac{2\pi}{5}\right)$$

b. $4\cos^2 35° - 2$

$$= 2(2\cos^2 35° - 1)$$
$$= 2\cos 2(35°)$$
$$= 2\cos(70°)$$

c. $\sin 60° \cos 60°$

$$= \frac{1}{2}(2\sin 60° \cos 60°)$$
$$= \frac{1}{2}\sin 2(60°)$$
$$= \frac{1}{2}\sin(120°)$$

3. If $\cos A = {}^{5}/_{7}$ and $\measuredangle A$ is in the fourth quadrant, find the exact value of $\sin 2A$.

$$x^2 + y^2 = r^2$$
$$(5)^2 + y^2 = (7)^2$$
$$25 + y^2 = 49$$
$$y^2 = 24$$
$$y = -\sqrt{24}$$
$$y = -2\sqrt{6}$$

$$\sin A = -\frac{2\sqrt{6}}{7}$$

$$\sin 2A = 2\sin A \cos A$$
$$= 2\left(-\frac{2\sqrt{6}}{7}\right)\left(\frac{5}{7}\right)$$
$$= -\frac{20\sqrt{6}}{49}$$

4. Prove the following equation.

$$\frac{\sin 3x}{\sin x \cos x} = 4\cos x - \sec x$$

$\frac{\sin 3x}{\sin x \cos x}$	$4\cos x - \sec x$
$= \frac{\sin(2x + x)}{\sin x \cos x}$	$= 4\cos x - \frac{1}{\cos x}$
$\sin(A + B) = \sin A \cos B + \cos A \sin B$ $(A = 2x, B = x)$ Angle Sum Identity	$= \frac{4\cos^2 x - 1}{\cos x}$
$= \frac{\sin 2x \cos x + \cos 2x \sin x}{\sin x \cos x}$	
$= \frac{(2\sin x \cos x)\cos x + (2\cos^2 A - 1)\sin x}{\sin x \cos x}$ Double Angle Identities	
$= \frac{2\sin x \cos^2 x + 2\sin x \cos^2 x - \sin x}{\sin x \cos x}$ Expand	
$= \frac{4\sin x \cos^2 x - \sin x}{\sin x \cos x}$	
$= \frac{\sin x\,(4\cos^2 x - 1)}{\sin x \cos x}$	
$= \frac{4\cos^2 x - 1}{\cos x}$	

Assignment p. 306 # 1(ce), 2(c), 3 – 5, 8(bdf), 11 – 18, 21, 24, C1 Quiz Next Day!