Name: ___________________________ Date: ____________________

Learning Goal 8.1	Solving exponential and logarithmic equations with same base and with different bases, including base e.

Power Law	Product Law	Quotient Law
$\log_b x^n = n\log_b x$	$\log_b(xy) = \log_b x + \log_b y$	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

Example Evaluate.

a. $\log_5 75 - \log_5 3$

Quotient

$= \log_5\left(\frac{75}{3}\right)$

$= \log_5(25)$

$= 2$

$\log_5 75 - \log_3 3$

DON'T MATCH ⇒ DON'T USE THE QUOTIENT LAW.

b. $\log_2 8 = \log_2(2^3)$

$= 3\log_2 2$

$= 3$

Example Simplify.

a. $5^{\log_5(a+b)}$

$x \times 2 \div 2$

* IF THE EXP. AND THE LOGARITHM ARE THE SAME BASE, THEY CANCEL.

$= a+b$

b. $8^{2\log_2 m - {}^1/_2 \log_2 n^6}$

* DON'T MATCH DON'T CANCEL

$= (2^3)^{2\log_2 m}$

$= 2^{6\log_2 m}$

$= 2^{\log_2 m^6}$

$= m^6$

POWER

$= (2^3)^{\log_2 m^2 - \log_2 n^3}$

$= (2^3)^{\log_2\left(\frac{m^2}{n^3}\right)}$

$= 2^{3\log_2\left(\frac{m^2}{n^3}\right)}$

POWER

$= 2^{\log_2\left(\frac{m^2}{n^3}\right)^3}$

$= \frac{m^6}{n^9}$

Example Change of Base

$$\log_b x = \frac{\log_a x}{\log_a b}$$

$\log_5 3 = \frac{\log_2 3}{\log_2 5}$

log

KIND OF OBSOLETE NOW THAT WE BETTER CALCULATORS

Example Write as a single logarithm.

a. $\dfrac{\log_{11} 10}{\log_{11} 5} = \log_5 10$

b. $\dfrac{\log_3 7}{\log_3 4} = \log_4 7$

Example Simplify by changing the base of the logarithm. Check using a calculator.

a. $\log_{27} 9$

$= \dfrac{\log_3 9}{\log_3 27}$

$= \dfrac{2}{3}$

$(\sqrt[3]{27})^2 = 9$

$(27)^{2/3} = 9$

b. $\log_8 32 + \log_{16} 2$

DON'T MATCH
⇒ DON'T USE PRODUCT

$= \dfrac{\log_2 32}{\log_2 8} + \dfrac{\log_2 2}{\log_2 16}$

$= \dfrac{5}{3} + \dfrac{1}{4} = \dfrac{23}{12}$

Example Simplify. State any restrictions on the variable.

RESTRICTIONS:
$x > 0$

$$4\log_3 x - \frac{1}{2}(\log_3 x + 5\log_3 x)$$

POWER

$= 4\log_3 x - \frac{1}{2}(\log_3 x + \log_3 x^5)$

PRODUCT

$= 4\log_3 x - \frac{1}{2}(\log_3 x^6)$

POWER

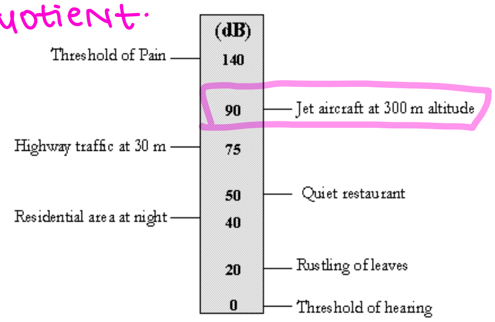
$= 4\log_3 x - \log_3 x^3$

POWER

$= \log_3 x^4 - \log_3 x^3 = \log_3\left(\dfrac{x^4}{x^3}\right) = \log_3 x$

QUOTIENT

Example The decibel scale measures the loudness of sound. Each 10 unit step on the scale represents a 10 fold increase in loudness. The intensity level, β is related to I, the intensity of the sound, in watts per square metre $\left({}^{\mathrm{W}}/_{\mathrm{m}^2}\right)$ and $I_0 = 10^{-12}\ {}^{\mathrm{W}}/_{\mathrm{m}^2}$ (the faintest sound that can be heard by a person with normal hearing) by the following:



x = SAFE
W = WHISPER

$$\beta = 10\log\left(\frac{\mathrm{I}}{\mathrm{I}_0}\right)$$

Sounds that are at most 100 000 times as intense as a whisper are considered to be safe, no matter how long or how often you hear them. The sound level of a whisper is 20 dB. What sound level can be considered safe, no matter how long it lasts?

$\dfrac{I_x}{I_W} = 100\ 000$

$\beta_x - \underset{20}{\beta_W} = 10\log\left(\dfrac{I_x}{I_0}\right) - 10\log\left(\dfrac{I_W}{I_0}\right)$

$= 10\left(\log\left(\dfrac{I_x}{I_0}\right) - \log\left(\dfrac{I_W}{I_0}\right)\right)$

Assignment p. 400 # 11 – 16, 18 – 20, C2 Quiz Next Day!

$= 10\left(\log\left(\dfrac{I_x}{\cancel{I_0}} \times \dfrac{\cancel{I_0}}{I_W}\right)\right)$

You can listen to 70 dB for any length of time without hearing damage.

$$= 10 \log\left(\frac{I_x}{I_w}\right)$$

$$= 10 \log_{10}(100\,000)$$

$$= 10(5)$$

$$\beta_x - 20 = 50$$

$$\beta_x = 70 \text{ dB}$$