

Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Learning Goal 8.1**Solving exponential and logarithmic equations with same base and with different bases, including base  $e$ .**More Questions - Solutions**

Power Law	Product Law	Quotient Law	Change of Base
$\log_b x^y = y \log_b x$	$\log_b(xy) = \log_b x + \log_b y$	$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$	$\log_b x = \frac{\log_a x}{\log_a b}$

1. Evaluate.

a.  $\log_{36} 2 - \log_{36} 12$

$$\begin{aligned}
 &= \log_{36} \left(\frac{2}{12}\right) \\
 &= \log_{36} \left(\frac{1}{6}\right) \\
 &= -\frac{1}{2}
 \end{aligned}$$

b.  $2 \log_3 6 - \frac{1}{2} \log_3 64 + \log_3 2$

$$\begin{aligned}
 &= \log_3 6^2 - \log_3 \sqrt{64} + \log_3 2 \\
 &= \log_3 36 - \log_3 8 + \log_3 2 \\
 &= \log_3 \left(\frac{36}{8}\right) + \log_3 2 \\
 &= \log_3 \left(\frac{9}{2}\right) + \log_3 2 \\
 &= \log_3 \left(\frac{9}{2} \times 2\right) \\
 &= \log_3(9) \\
 &= 2
 \end{aligned}$$

2. Write as a single logarithm.

a.  $\frac{n \log_a x}{\log_a y}$

$$\begin{aligned}
 &= \frac{\log_a x^n}{\log_a y} \\
 &= \log_y x^n
 \end{aligned}$$

b.  $\frac{\log_6 64}{\log_6 4}$

$$\begin{aligned}
 &= \log_4 64 \\
 &= 3
 \end{aligned}$$

3. Simplify by changing the base of the logarithm. Check using a calculator.

a.  $\log_{125} 625$

$$\begin{aligned} &= \frac{\log_5 625}{\log_5 125} \\ &= \frac{4}{3} \end{aligned}$$

b.  $\log_8 32 + \log_{16} 2 - \log_2 4$

$$\begin{aligned} &= \frac{\log_2 32}{\log_2 8} + \frac{\log_2 2}{\log_2 16} - \log_2 4 \\ &= \frac{5}{3} + \frac{1}{4} - 2 \\ &= \frac{20}{12} + \frac{3}{12} - \frac{24}{12} \\ &= -\frac{1}{12} \end{aligned}$$

4. Simplify. State any restrictions on the variable.

$$\begin{aligned} &\log_2(x^2 - 9) - \log_2(x^2 - x - 6) \\ &= \log_2\left(\frac{x^2 - 9}{x^2 - x - 6}\right) \\ &= \log_2\left(\frac{(x + 3)(x - 3)}{(x - 3)(x + 2)}\right) \quad x \neq -2, 3 \\ &= \log_2\left(\frac{x + 3}{x + 2}\right) \end{aligned}$$

5. Audiologists recommend hearing protection if the sound level in environment exceeds 85 dB. The sound level of a chainsaw is about 85 dB and the maximum level of a AirPods is about 110 dB. How times as intense is the sound of the media player, at the maximum volume, compared to the sound of a chainsaw?

Let  $\beta_A$  = the decibel level of the AirPods and  $\beta_C$  = the decibel level of the chainsaw.

$$\beta_A - \beta_C = 10 \log\left(\frac{I_A}{I_0}\right) - 10 \log\left(\frac{I_C}{I_0}\right)$$

$$\beta_A - \beta_C = 10 \left( \log\left(\frac{I_A}{I_0}\right) - \log\left(\frac{I_C}{I_0}\right) \right)$$

$$\beta_A - \beta_C = 10 \left( \log\left(\frac{I_A}{I_0} \div \frac{I_C}{I_0}\right) \right)$$

$$\beta_A - \beta_C = 10 \left( \log\left(\frac{I_A}{I_0} \times \frac{I_0}{I_C}\right) \right)$$

$$\beta_A - \beta_C = 10 \left( \log\left(\frac{I_A}{I_C}\right) \right)$$

$$110 - 85 = 10 \left( \log\left(\frac{I_A}{I_C}\right) \right)$$

$$25 = 10 \left( \log\left(\frac{I_A}{I_C}\right) \right)$$

$$2.5 = \log\left(\frac{I_A}{I_C}\right)$$

$$10^{2.5} = \frac{I_A}{I_C}$$

$$316 \approx \frac{I_A}{I_C}$$

AirPods are over 300 times more intense than a chainsaw when at maximum volume. Turn them down!