

Name: _____

Date: _____

Learning Goal 0.2

Expectations for algebra from previous years.

Example Factor the following expressions.

a. $x^2 + 5x - 24$

$$= (x+8)(x-3)$$

b. $x^4 - 16$

$$= \underbrace{(x^2 - 4)}_{\text{DOS}}(x^2 + 4)$$
$$= (x+2)(x-2)(x^2 + 4)$$

c. $x^3 - 7x^2 + 12x$

$$= x(x^2 - 7x + 12)$$
$$= x(x-4)(x-3)$$

d. $12x^5 - 19x^3 - 18x$

$$= x(12x^4 - 19x^2 - 18)$$
$$= x(12x^4 + 8x^2 - 27x^2 - 18)$$
$$= x[4x^2(3x^2 + 2) - 9(3x^2 + 2)]$$
$$= x(3x^2 + 2)\underbrace{(4x^2 - 9)}_{\text{DOS}}$$
$$= x(3x^2 + 2)(2x + 3)(2x - 3)$$

e. $y^2 - x^2 + 6x - 9$

$$= y^2 - (x^2 - 6x + 9)$$
$$= y^2 - (x-3)^2$$
$$= (y + (x-3))(y - (x-3))$$
$$= (y + x - 3)(y - x + 3)$$

f. $x^4 + 4$

↳ Not Factorable

g. $x^6 - 8y^3$

FORMULA:

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

$$a = x^2, b = 2y$$

$$= (x^2 - 2y)(x^4 + 2x^2y + 4y^2)$$

h. $\frac{128}{\sqrt[3]{x}} - 2x^{17/3} = \frac{1}{\sqrt[3]{x}}(128 - 2x^6)$

$$= \frac{2}{\sqrt[3]{x}}(64 - x^6) \quad \begin{matrix} a = 4 \\ b = x^2 \end{matrix}$$

$$= \frac{2}{\sqrt[3]{x}}\underbrace{(4 - x^2)}_{\text{DOS}}(16 + 4x^2 + x^4)$$

$$= \frac{2}{\sqrt[3]{x}}(2+x)(2-x)(16 + 4x^2 + x^4) \times \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$$

$$= \frac{2\sqrt[3]{x^2}}{x}(2+x)(2-x)(16 + 4x^2 + x^4)$$

Example Simplify the following rational expressions using factoring.

$$\begin{aligned} \text{a. } & \frac{a + \frac{a}{b}}{1 + \frac{1}{b}} \\ & = \frac{\cancel{\frac{1}{b}}(ab + a)}{\cancel{\frac{1}{b}}(b+1)} \\ & = \frac{a(\cancel{b+1})}{\cancel{b+1}} \\ & = a \end{aligned}$$

NVPS: $b \neq 0, -1$

$$\begin{aligned} \text{b. } & \frac{x^2 - \frac{1}{x}}{x^3 - \frac{1}{x^3}} \\ & = \frac{x^{-1} \cancel{\frac{1}{x}}(x^3 - 1)}{x^{-3} \cancel{\frac{1}{x^3}}(x^3 - 1)} \\ & = \frac{x^3(x^3 - 1)}{x(x^3 - 1)} \\ & = \frac{x^2(x^3 - 1)}{x^3 - 1} \end{aligned}$$

$$\begin{aligned} & = \frac{x^2(x-1)(x^2+x+1)}{(x^2-1)(x^4+x^2+1)} \\ & = \frac{x^2(\cancel{x-1})(x^2+x+1)}{(x+1)(\cancel{x-1})(x^4+x^2+1)} \\ & = \frac{x^2(x^2+x+1)}{(x+1)(x^4+x^2+1)} \end{aligned}$$

NVPS: $x \neq 0, \pm 1$

$$\begin{aligned} \text{c. } & \frac{x + 7 - \frac{8}{x}}{x + 10 + \frac{16}{x}} \\ & = \frac{\cancel{\frac{1}{x}}(x^2 + 7x - 8)}{\cancel{\frac{1}{x}}(x^2 + 10x + 16)} \\ & = \frac{(\cancel{x+8})(x-1)}{(\cancel{x+8})(x+2)} \\ & = \frac{x-1}{x+2} \end{aligned}$$

NVPS: $x \neq 0, -2, -8$