

Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Learning Goal 0.2**

Expectations for algebra from previous years.

**More Questions – Solutions**

1. Fully factor the following expressions.

$$\begin{aligned} \text{a. } x^2 - 18x + 72 \\ = (x - 12)(x - 6) \end{aligned}$$

$$\begin{aligned} \text{b. } x^4 + 2x^2 - 24 \\ = (x^2 + 6)(x^2 - 4) \\ = (x^2 + 6)(x + 2)(x - 2) \end{aligned}$$

$$\begin{aligned} \text{c. } m^2 - 14m + 24 \\ = (m - 12)(m - 2) \end{aligned}$$

$$\begin{aligned} \text{d. } 36x^2 + 12x + 1 \\ = (6x + 1)^2 \end{aligned}$$

$$\begin{aligned} \text{e. } 144 - n^8 \\ = (12 + n^4)(12 - n^4) \end{aligned}$$

$$\begin{aligned} \text{f. } 20 + 8n - n^2 \\ = -(n^2 - 8n - 20) \\ = -(n - 10)(n + 2) \end{aligned}$$

$$\begin{aligned} \text{g. } x^2 + 11x - 80 \\ = (x + 16)(x - 5) \end{aligned}$$

$$\begin{aligned} \text{h. } 11p - p^2 - 24 \\ = -(p^2 - 11p + 24) \\ = -(p - 8)(p - 3) \end{aligned}$$

$$\begin{aligned} \text{i. } 6y^2 + 5y - 6 \\ = 6y^2 + 9y - 4y - 6 \\ = 3y(2y + 3) - 2(2y + 3) \\ = (2y + 3)(3y - 2) \end{aligned}$$

$$\begin{aligned} \text{j. } 24x^2 - 20x - 24 \\ = 4(6x^2 - 5x - 6) \\ = 4(6x^2 - 9x + 4x - 6) \\ = 4(3x(2x - 3) + 2(2x - 3)) \\ = 4(2x - 3)(3x + 2) \end{aligned}$$

$$\begin{aligned} \text{k. } 162v^4 - 2w^4 \\ = 2(81v^4 - w^4) \\ = 2(9v^2 - w^2)(9v^2 + w^2) \\ = 2(3v + w)(3v - w)(9v^2 + w^2) \end{aligned}$$

$$\begin{aligned} \text{l. } 21 + 66x + 9x^2 \\ = 9x^2 + 63x + 3x + 21 \\ = 9x(x + 7) + 3(x + 7) \\ = (x + 7)(9x + 3) \end{aligned}$$

$$\begin{aligned} \text{m. } 2x^2 + 5xy + 2y^2 \\ = 2x^2 + 4xy + xy + 2y^2 \\ = 2x(x + 2y) + y(x + 2y) \\ = (x + 2y)(2x + y) \end{aligned}$$

$$\begin{aligned} \text{n. } 16b^2 + 60b - 100 \\ = 4(4b^2 + 15b - 25) \\ = 4(4b^2 + 20b - 5b - 25) \\ = 4(4b(b + 5) - 5(b + 5)) \\ = 4(b + 5)(4b - 5) \end{aligned}$$

$$\begin{aligned} \text{o. } 4b^2 - 35ab + 49a^2 \\ = 4b^2 - 28ab - 7ab + 49a^2 \\ = 4b(b - 7a) - 7a(b - 7a) \\ = (b - 7a)(4b - 7a) \end{aligned}$$

$$\begin{aligned} \text{p. } 8x^2 - 26x + 15 \\ = 8x^2 - 20x - 6x + 15 \\ = 4x(2x - 5) - 3(2x - 5) \\ = (2x - 5)(4x - 3) \end{aligned}$$

$$\begin{aligned} \text{q. } 6x^2 - 17x + 5 \\ = 6x^2 - 15x - 2x + 5 \\ = 3x(2x - 5) - (2x - 5) \\ = (2x - 5)(3x - 1) \end{aligned}$$

$$\begin{aligned} \text{r. } 4x^4 - 21x^2 - 18 \\ = 4x^2 + 3x - 24x - 18 \\ = x(4x + 3) - 6(4x + 3) \\ = (4x + 3)(x - 6) \end{aligned}$$

$$\begin{aligned} \text{s. } & 27x^3 - y^6 \\ & a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2) \\ & = (3x - y^2)(9x^2 + 3xy^2 + y^4) \end{aligned}$$

$$\begin{aligned} \text{t. } & (x + 2)^3 + (x - 2)^3 \\ & a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2) \\ & = ((x + 2) + (x - 2)) \times \\ & \quad ((x + 2)^2 - (x + 2)(x - 2) + (x - 2)^2) \\ & = 2x((x^2 + 4x + 4) - (x^2 - 4) \\ & \quad + (x^2 - 4x + 4)) \\ & = 2x(x^2 + 12) \end{aligned}$$

$$\begin{aligned} \text{u. } & 2xy - 2xz - 3y^2 + 3yz \\ & = 2x(y - z) - 3y(y - z) \\ & = (y - z)(2x - 3y) \end{aligned}$$

Or expand, collect like terms and factor!

$$\begin{aligned} \text{v. } & x^2 + 6x + 9 - 4y^2 \\ & = (x + 3)^2 - 4y^2 \\ & = ((x + 3) + 2y)((x + 3) - 2y) \end{aligned}$$

$$\begin{aligned} \text{w. } & x^{5/2} - 3x^{3/2} + 2x^{1/2} \\ & = x^{1/2}(x^2 - 3x - 2) \\ & = x^{1/2}(x - 2)(x + 1) \end{aligned}$$

$$\begin{aligned} \text{x. } & x^{20/3} + x^{11/3} - 2x^{2/3} \\ & = x^{2/3}(x^6 + x^3 - 2) \\ & = x^{2/3}(x^3 + 2)(x^3 - 1) \\ & = x^{2/3}(x^3 + 2)(x - 1)(x^2 + x + 1) \end{aligned}$$

$$\begin{aligned} \text{y. } & 4x^{1/3} - 3x^{-2/3} - x^{-5/3} \\ & = x^{-5/3}(4x^2 - 3x - 1) \\ & = x^{-5/3}(4x^2 - 4x + x - 1) \\ & = x^{-5/3}(4x(x - 1) + (x - 1)) \\ & = x^{-5/3}(x - 1)(4x + 1) \end{aligned}$$

$$\begin{aligned} \text{z. } & 4x^2y^4 - 36x^4y^2 \\ & = 4x^2y^2(y^2 - 9x^2) \\ & = 4x^2y^2(y + 3x)(y - 3x) \end{aligned}$$

2. Simplify the following rational expressions using factoring. State any restrictions on the domain.

$$\begin{aligned} \text{a. } & \frac{x - 3 + \frac{2}{x}}{x - 4 + \frac{3}{x}} \\ & = \frac{\frac{1}{x}(x^2 - 3x + 2)}{\frac{1}{x}(x^2 - 4x + 3)} \\ & = \frac{\frac{1}{x}(x - 2)(x - 1)}{\frac{1}{x}(x - 3)(x - 1)} \\ & = \frac{x - 2}{x - 3} \end{aligned}$$

$$x \neq 0, 1, 3$$

$$\begin{aligned} \text{b. } & \frac{\frac{x^3 + 27}{x^2 - 9}}{\frac{x^2 - 3x + 9}{x + 3}} \\ & a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2) \\ & = \frac{(x + 3)(x^2 - 3x + 9)}{(x + 3)(x - 3)} \\ & = \frac{x^2 - 3x + 9}{x - 3} \\ & = \frac{x^2 - 3x + 9}{x - 3} \times \frac{x + 3}{x^2 - 3x + 9} \\ & = \frac{x + 3}{x - 3} \end{aligned}$$

$$x \neq \pm 3$$

$$\begin{aligned} \text{c. } & \frac{\frac{1}{x - y} - \frac{1}{x + y}}{\frac{2}{x^2 - y^2}} \\ & = \frac{\frac{x + y}{x^2 - y^2} - \frac{x - y}{x^2 - y^2}}{\frac{2}{x^2 - y^2}} \\ & = \frac{2y}{x^2 - y^2} \\ & = \frac{2y}{x^2 - y^2} \times \frac{x^2 - y^2}{2} \\ & = y \end{aligned}$$

$$x \neq \pm y$$

$$\begin{aligned} \text{d. } & x - \frac{1+x}{1-x^2} \\ &= x - \frac{1+x}{(1+x)(1-x)} \\ &= x - \frac{1}{1-x} \\ &= \frac{x(1-x) - 1}{1-x} \\ &= \frac{x - x^2 - 1}{1-x} \\ &= \frac{-(x^2 - x + 1)}{-(x-1)} \\ &= \frac{x^2 - x + 1}{x-1} \end{aligned}$$

$$x \neq \pm 1$$