

Name: \_\_\_\_\_

Date: \_\_\_\_\_

**Learning Goal 5.2**

Use exponent laws to evaluate expression with positive and negative rational exponents.

Recall the Product of Powers rule to simplify the following expressions.

$$\begin{aligned} \text{a. } 3^6 3^2 \\ &= 3^{6+2} \\ &= 3^8 \end{aligned}$$

$$\begin{aligned} \text{b. } 6^7 6^2 \\ &= 6^{7+2} \\ &= 6^9 \end{aligned}$$

$$\begin{aligned} \text{c. } 7^6 7^1 \\ &= 7^{6+1} \\ &= 7^7 \end{aligned}$$

And the Quotient of Powers rule:

$$\begin{aligned} \text{d. } 3^6 \div 3^2 \\ &= 3^{6-2} \\ &= 3^4 \end{aligned}$$

$$\begin{aligned} \text{e. } \frac{6^2}{6^7} &= 6^{2-7} \\ &= 6^{-5} \end{aligned}$$

$$\begin{aligned} \text{f. } 7^1 \div 7^6 &= 7^{1-6} \\ &= 7^{-5} \end{aligned}$$

Take a (silent) minute. What do you think the negative exponents represent?

$$= \frac{6^1 \times 6^1}{6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6} = \frac{1}{6^5}$$

Negative exponents are related to reciprocals

- \* if the -ve exponent is in the numerator, it moves to the denominator
- \* if the -ve exponent is in the denominator, it moves to the numerator.

**Example** Evaluate the following expressions **without a calculator**. Leave your answers as fractions.

$$\begin{aligned} \text{a. } 7^{-2} &= \frac{1}{7^2} \\ &= \frac{1}{49} \end{aligned}$$

$$\begin{aligned} \text{b. } (-1.5)^{-3} &= \frac{1}{(-1.5)^3} \\ &= \left(-\frac{3}{2}\right)^{-3} = \left(-\frac{2}{3}\right)^3 = \frac{(-2)^3}{(3)^3} = \frac{-8}{27} \end{aligned}$$

$$\begin{aligned} \text{c. } \left(-\frac{3}{4}\right)^{-3} &= \left(-\frac{4}{3}\right)^3 = -\frac{4^3}{3^3} = -\frac{64}{27} \end{aligned}$$

$$\begin{aligned} \text{d. } \left(\frac{10}{3}\right)^{-2} &= \left(\frac{3}{10}\right)^2 = \frac{3^2}{10^2} = \frac{9}{100} \end{aligned}$$

**Example** Simplify the following expressions to a single power with only positive exponents. Do not evaluate. Show all your work.

$$\begin{aligned} \text{a. } -((3^2 \times 3^{-7})^{-2})^2 &= -((3^{-5})^{-2})^2 \\ &= -(3^{10})^2 \\ &= -3^{20} \end{aligned}$$

$$\begin{aligned} \text{b. } -(q^{-5} \times q^{-4})^2)^{-4} &= -((q^{-9})^2)^{-4} \\ &= -\left(\frac{1}{q^9}\right)^{-4} \\ &= \left(-\frac{1}{q^{18}}\right)^{-4} \end{aligned}$$

$-3 \neq \frac{1}{3}$

$$\begin{aligned} &= \frac{1}{q^{-72}} \\ &= q^{72} \end{aligned}$$

$$\begin{aligned} \text{c. } \left(-\frac{x^{-5}}{x^2}\right)^{-4} &= \left(-\frac{1}{x^5 x^2}\right)^{-4} \\ &= \left(-\frac{1}{x^7}\right)^{-4} \\ &= (-x^7)^4 \\ &= x^{28} \end{aligned}$$

$$\begin{aligned} \text{d. } \left(\frac{64}{32^{-2}}\right)^{-4} &= \left(\frac{2^6}{(2^5)^{-2}}\right)^{-4} \\ &= \left(\frac{2^6}{2^{-10}}\right)^{-4} \\ &= (2^6 \times 2^{10})^{-4} \\ &= (2^{16})^{-4} \\ &= \left(\frac{1}{2^{16}}\right)^4 \\ &= \frac{1}{2^{64}} \end{aligned}$$