Name: ________________________ Date: ___________________

Learning Goal 2.1	Finite limits and continuity.

We will apply these methods to **four** different types of limits:

1. Basic 2. one - sided 3. infinite 4. limits at infinity

we've done numerically + graphically, now algebraically

Example Compute the value of the following limits.

a. $\lim_{x\to-2} 3x^2 + 5x - 9 = 3(-2)^2 + 5(-2) - 9 = -7$

$= \lim_{x\to-2} 3x^2 + \lim_{x\to-2} 5x - \cancel{\lim_{x\to-2}} 9$

$= 3\lim_{x\to-2} x^2 + 5\lim_{x\to-2} x - 9$ $\quad -2$

$= 3\left(\lim_{x\to-2} x\right)^2 + 5(-2) - 9 = 3(-2)^2 - 10 - 9$

$= 12 - 10 - 9$

$= -7$

b. $\lim_{x\to1} \frac{6 - 3x + 10x^2}{2x^4 + 7x^3 + 1}$

$= \frac{\lim_{x\to1} 6 - 3x + 10x^2}{\lim_{x\to1} 2x^4 + 7x^3 + 1} = \frac{6 - 3 + 10}{2 + 7 + 1}$

$= \frac{13}{10}$

c. $\lim_{x\to1} \frac{x^2 + 1}{x - 1}$ $\quad f(x)$, $g(x)$

$= \frac{\text{polynomial}}{\text{polynomial}}$ } both smooth and continuous

$= \frac{f(1)}{g(1)}$

$= \frac{2}{0} = \text{DNE}$

$= \text{undefined}$

d. $\lim_{x\to1} \frac{x^2 - 1}{x - 1} = \frac{(1)^2 - 1}{(1) - 1}$

$= \frac{0}{0}$ } indeterminate.

$= \lim_{x\to1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$

$= \lim_{x\to1} x + 1 = 2$

Undefined vs. Indeterminate

DNE $\Rightarrow$ DONE

$\frac{0}{0} \Rightarrow$ more work to be done.

e. $\lim_{x\to2} \frac{x^2 + 4x - 12}{x^2 - 2x}$

$= \frac{(2)^2 + 4(2) - 12}{(2)^2 - 2(2)}$

$= \frac{0}{0}$ INDETERMINANT.

$= \lim_{x\to2} \frac{(x+6)\cancel{(x-2)}}{x\cancel{(x-2)}}$

$= \frac{(2) + 6}{(2)} = 4$

f. $\lim_{x\to0} \frac{2(-3 + x)^2 - 18}{x} = \frac{0}{0}$

$= \lim_{x\to0} \frac{2(9 - 6x + x^2) - 18}{x}$

$= \lim_{x\to0} \frac{\cancel{18} - 12x + 2x^2 - \cancel{18}}{x}$

$= \lim_{x\to0} \frac{2x^2 - 12x}{x}$

$= \lim_{x\to0} 2x - 12$

$= -12$

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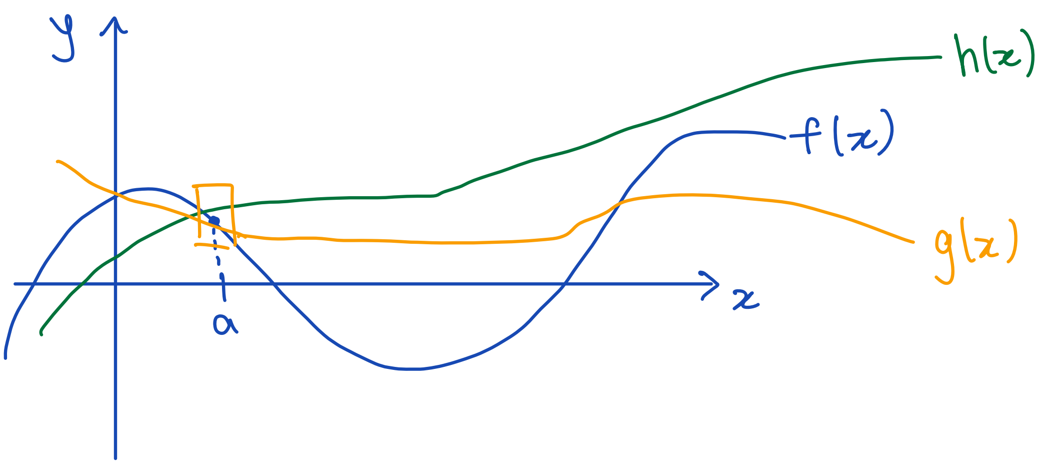
g. $\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2} \times \frac{(\sqrt{x^2+9}+3)}{\sqrt{x^2+9}+3}$ $\sqrt{x^2+9} \neq x+3$

$= \lim_{x\to 0} \frac{(x^2+9)+\cancel{3\sqrt{x^2+9}}-\cancel{3\sqrt{x^2+9}}-9}{x^2(\sqrt{x^2+9}+3)}$

$= \lim_{x\to 0} \frac{\cancel{x^2}^{\,1}}{\cancel{x^2}(\sqrt{x^2+9}+3)}$

$= \frac{1}{\sqrt{0+9}+3} = \frac{1}{6}$

The Squeeze Theorem if $g(x) \le f(x) \le h(x)$ when $x \approx a$ and $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$ then $\lim_{x\to a} f(x) = L$.



Example Evaluate.

$$\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right)$$

$-1 \le \cos\left(\frac{1}{x}\right) \le 1$

$-x^2 \le x^2\cos\left(\frac{1}{x}\right) \le x^2$

$\lim_{x\to 0} -x^2 = 0$ $0 \le$ ≤ 0 $\lim_{x\to 0} x^2 = 0$

$\lim_{x\to 0} x^2\cos\left(\frac{1}{x}\right) = 0$

Assignment # 13, 11 – 33, 43 – 51, 48 Quiz Next Day!