Name: ________________________ Date: __________________

| Learning Goal 2.1 | Finite limits and continuity. |
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**More Questions – Solutions**

1. Compute the value of the following limits.

a.
$$\lim_{x\to 1}\frac{x^2+2x-3}{x-1}$$
$$=\frac{0}{0}$$
$$=\lim_{x\to 1}\frac{(x+3)(x-1)}{x-1}$$
$$=\lim_{x\to 1}x+3$$
$$=1+3$$
$$=4$$

b.
$$\lim_{x\to -1}\frac{\sqrt{x+5}-2}{x+1}$$
$$=\frac{0}{0}$$
$$=\lim_{x\to -1}\frac{\sqrt{x+5}-2}{x+1}\times\frac{\sqrt{x+5}+2}{\sqrt{x+5}+2}$$
$$=\lim_{x\to -1}\frac{(x+5)-4}{(x+1)(\sqrt{x+5}+2)}$$
$$=\lim_{x\to -1}\frac{x+1}{(x+1)\left(\sqrt{x+5}+2\right)}$$
$$=\lim_{x\to -1}\frac{1}{\sqrt{x+5}+2}$$
$$=\frac{1}{\sqrt{(-1)+5}+2}$$
$$=\frac{1}{4}$$

c.
$$\lim_{x\to 1}\frac{x^3-1}{x-1}$$
$$=\frac{0}{0}$$
$$=\lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{x-1}$$
$$=\lim_{x\to 1}x^2+x+1$$
$$=(1)^2+(1)+1$$
$$=3$$

d.
$$\lim_{x\to 0}\frac{x^3-x^2}{x^2}$$
$$=\frac{0}{0}$$
$$=\lim_{x\to 0}\frac{x^2(x-1)}{x^2}$$
$$=\lim_{x\to 0}x-1$$
$$=(0)-1$$
$$=-1$$

e. $\lim_{x\to 1}\frac{x^2}{x(x-1)}$

$$=\frac{1}{0}$$

$$\lim_{x\to 1^-}\frac{x^2}{x(x-1)} = \frac{(+)^2}{(+)(-)} = \frac{+}{-} = -\infty$$

$$\lim_{x\to 1^+}\frac{x^2}{x(x-1)} = \frac{(+)^2}{(+)(+)} = \frac{+}{+} = \infty$$

$$\lim_{x\to 1}\frac{x^2}{x(x-1)} = \text{DNE}$$

And there is a vertical asymptote at $x = 1$.

f. $\lim_{x\to 1}\frac{x-1}{\sqrt{x}-1}$

$$=\frac{0}{0}$$
$$=\lim_{x\to 1}\frac{x-1}{\sqrt{x}-1}\times\frac{\sqrt{x}+1}{\sqrt{x}+1}$$
$$=\lim_{x\to 1}\frac{(x-1)(\sqrt{x}+1)}{x-1}$$
$$=\lim_{x\to 1}\sqrt{x}+1$$
$$=\sqrt{(1)}+1$$
$$=2$$

g. $\lim_{x\to -1}\frac{x+1}{x^3+1}$

$$=\frac{0}{0}$$
$$=\lim_{x\to -1}\frac{x+1}{(x+1)(x^2-x+1)}$$
$$=\lim_{x\to -1}\frac{1}{x^2-x+1}$$
$$=\frac{1}{(-1)^2-(-1)+1}$$
$$=\frac{1}{3}$$

h. $\lim_{x\to 1}\frac{x^3-3x+2}{x^3-x^2-x+1}$

$$=\frac{0}{0}$$
$$=\lim_{x\to 1}\frac{x^3-3x+2}{x^3-x^2-x+1}$$
$$=\lim_{x\to 1}\frac{(x+2)(x-1)^2}{(x+1)(x-1)^2}$$
$$=\lim_{x\to 1}\frac{x+2}{x+1}$$
$$=\frac{(1)+2}{(1)+1}$$
$$=\frac{3}{2}$$

i.

$$\lim_{x\to 0}\frac{x}{\sqrt{x+1}-1}$$
$$=\frac{0}{0}$$
$$=\lim_{x\to 0}\frac{x}{\sqrt{x+1}-1}\times\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$$
$$=\lim_{x\to 0}\frac{x\left(\sqrt{x+1}+1\right)}{(x+1)-1}$$
$$=\lim_{x\to 0}\frac{x\left(\sqrt{x+1}+1\right)}{x}$$
$$=\lim_{x\to 0}\sqrt{x+1}+1$$
$$=\sqrt{(0)+1}+1$$
$$=2$$

j.

$$\lim_{x\to 1}\frac{\sqrt{x}-x^2}{1-\sqrt{x}}$$
$$=\frac{0}{0}$$
$$=\lim_{x\to 1}\frac{\sqrt{x}-x^2}{1-\sqrt{x}}\times\frac{\sqrt{x}+x^2}{\sqrt{x}+x^2}$$
$$=\lim_{x\to 1}\frac{x-x^4}{\sqrt{x}+x^2-x-x^2\sqrt{x}}$$
$$=\lim_{x\to 1}\frac{x(1-x^3)}{\sqrt{x}(1-x^2)-x(1-x)}$$
$$=\lim_{x\to 1}\frac{x(1-x^3)}{\sqrt{x}(1-x)(1+x)-x(1-x)}$$
$$=\lim_{x\to 1}\frac{-x(x^3-1)}{\sqrt{x}(1-x)(1+x)-x(1-x)}$$
$$=\lim_{x\to 1}\frac{-x(x-1)(x^2+x+1)}{-(x-1)\left(\sqrt{x}(1+x)-x\right)}$$
$$=\lim_{x\to 1}\frac{x(x^2+x+1)}{\sqrt{x}(1+x)-x}$$
$$=\frac{(1)\left((1)^2+(1)+1\right)}{\sqrt{(1)}\left(1+(1)\right)-(1)}$$
$$=3$$

k.

$$\lim_{x\to 0}\frac{\cos x-1}{x}$$
$$=\frac{0}{0}$$
$$=\lim_{x\to 0}\frac{\cos x-1}{x}\times\frac{\cos x+1}{\cos x+1}$$
$$=\lim_{x\to 0}\frac{\cos^2 x-1}{x(\cos x+1)}$$
$$=\lim_{x\to 0}\frac{-(1-\cos^2 x)}{x(\cos x+1)}$$
$$=\lim_{x\to 0}\frac{-\sin^2 x}{x(\cos x+1)}$$
$$=\lim_{x\to 0}-\frac{\sin x}{x}\times\frac{\sin x}{\cos x+1}$$
$$=\lim_{x\to 0}-\frac{\sin x}{x}\times\lim_{x\to 0}\frac{\sin x}{\cos x+1}$$
$$=(-1)\times\left(\frac{0}{2}\right)$$
$$=0$$

l.

$$\lim_{x\to 0}\frac{\sin 5x\cos x}{x}$$
$$=\frac{0}{0}$$
$$=\lim_{x\to 0}\frac{5\sin 5x\cos x}{5x}$$
$$=\lim_{x\to 0}\frac{\sin 5x}{5x}\times\lim_{x\to 0}5\cos x$$
$$=(1)\times(5\times 1)$$
$$=5$$

m.

$$\lim_{x\to 0}\frac{\tan^3 2x}{x^2\sin 7x}$$

$$=\frac{0}{0}$$

$$=\lim_{x\to 0}\frac{\tan^3 2x}{x^2\sin 7x}$$

$$=\lim_{x\to 0}\frac{\left(\frac{\sin 2x}{\cos 2x}\right)^3}{x^2\sin 7x}$$

$$=\lim_{x\to 0}\frac{(\sin 2x)^3}{x^2\sin 7x\,(\cos 2x)^3}$$

$$=\lim_{x\to 0}\frac{(2x)^3\times\left(\frac{\sin 2x}{2x}\right)^3}{x^2\left(\frac{\sin 7x}{7x}\right)\times 7x(\cos 2x)^3}$$

$$=\lim_{x\to 0}\frac{8x^3\left(\frac{\sin 2x}{2x}\right)^3}{7x^3\left(\frac{\sin 7x}{7x}\right)(\cos 2x)^3}$$

$$=\lim_{x\to 0}\frac{8\left(\frac{\sin 2x}{2x}\right)^3}{7\left(\frac{\sin 7x}{7x}\right)(\cos 2x)^3}$$

$$=\frac{8}{7}\times\frac{\left(\lim_{x\to 0}\frac{\sin 2x}{2x}\right)^3}{\left(\lim_{x\to 0}\frac{\sin 7x}{7x}\right)\left(\lim_{x\to 0}\cos 2x\right)^3}$$

$$=\frac{8}{7}\times\frac{(1)^3}{(1)(1)^3}$$

$$=\frac{8}{7}$$

n.

$$\lim_{x\to 0}x^3\cos\left(\frac{1}{\sqrt{x}}\right)$$

$$-1\le\cos\left(\frac{1}{\sqrt{x}}\right)\le 1$$

$$-1\times x^3\le x^3\times\cos\left(\frac{1}{\sqrt{x}}\right)\le 1\times x^3$$

$$-x^3\le x^3\cos\left(\frac{1}{\sqrt{x}}\right)\le x^3$$

$$\lim_{x\to 0}-x^3=0 \qquad \lim_{x\to 0}x^3=0$$

$$\lim_{x\to 0}x^3\cos\left(\frac{1}{\sqrt{x}}\right)=0$$