Name: ________________________ Date: __________________

Learning Goal 3.7	Creating confidence in word problems.

Play Day – Solutions

1. It is found that a certain manufacturer produces q thousand units per week when the unit price is $\$p$. Suppose the relationship between q and p is $q^2 - 3pq + p^2 = 5$. What is the rate of change of the supply when the quantity produced is 4 000 units and the unit price is $11, increasing at a rate of $0.10 per week?

NO Picture!! ☺

$\frac{dp}{dt} = 0.1$

$q = 4$

$p = 11$

$$q^2 \underbrace{- 3pq}_{\text{PRODUCT!}} + p^2 = 5$$

$$2q\frac{dq}{dt} - 3\left(q\frac{dp}{dt} + p\frac{dq}{dt}\right) + 2p\frac{dp}{dt} = 0$$

$$(2q - 3p)\frac{dq}{dt} - 3q\frac{dp}{dt} + 2p\frac{dp}{dt} = 0$$

$$(8 - 33)\frac{dq}{dt} = 3(4)(0.1) - 2(11)(0.1)$$

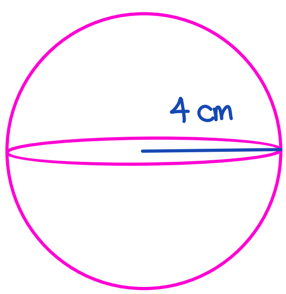
$$= 1.2 - 2.2$$

$$-25\frac{dq}{dt} = -1$$

$$\frac{dq}{dt} = \frac{1}{25} = 0.04$$

$\Rightarrow$ 40 units/week

2. You are inflating a spherical balloon at the rate of $7\ ^{\text{cm}^3}/_{\text{s}}$. How fast is the radius increasing when the radius is 4 cm?



$\frac{dV}{dt} = 7\ \text{cm}^3/\text{s}$

$\frac{dr}{dt} = ?$

$$V = \frac{4}{3}\pi r^3$$

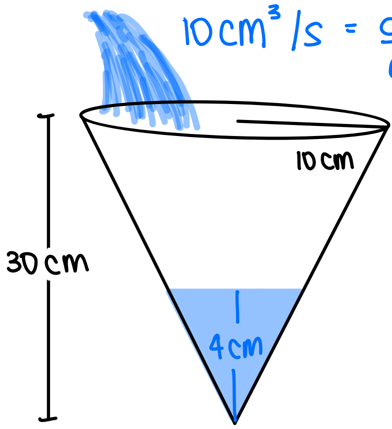
$$\frac{dV}{dt} = \frac{4}{3}\pi \times 3r^2 \times \frac{dr}{dt}$$

$$7 = \frac{4}{\cancel{3}}\pi \times \cancel{3}(4)^2 \times \frac{dr}{dt}$$

$$7 \times \frac{1}{4\pi} \times \frac{1}{4^2} = \frac{dr}{dt}$$

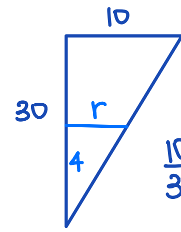
$$\frac{dr}{dt} = \frac{7}{4^3\pi} = \frac{7}{64\pi}\ \text{cm/s}$$

3. Water is poured into a conical container at the rate of $10\ \mathrm{cm}^3/\mathrm{s}$. The cone points directly down, and it has a height of 30 cm and a base radius of 10 cm. How fast is the water level rising when the water is 4 cm deep at its deepest point?



$10\,\mathrm{cm}^3/s = \frac{dV}{dt}$

$\frac{dh}{dt}$



$\frac{10}{30} = \frac{r}{h}$

$r = \frac{h}{3}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$

$= \frac{\pi h^3}{3^3}$

$\frac{dV}{dt} = \frac{\pi}{3^3} \times 3h^2 \times \frac{dh}{dt}$

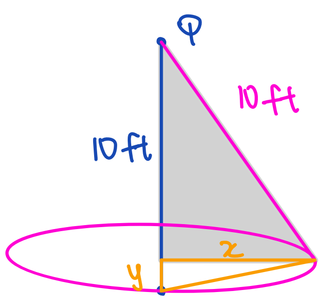
$10 = \frac{\pi}{3^{\cancel{3}\,2}} \times \cancel{3}(4)^2 \times \frac{dh}{dt}$

$10 \times \frac{3^2}{\pi} \times \frac{1}{4^2} = \frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{90}{16\pi}\ \mathrm{cm}/s$

4. A swing consists of a board a the end of a 10 ft long rope. Think of the board as a point P at the end fo the rope, and let Q be the point of attachment at the other end. Suppose that the swing is directly below Q at time $t = 0$, and is being pushed by someone who walks at $6\ \mathrm{ft}/\mathrm{s}$ from left to right.

a. How fast is the swing rising after 1 s?



$\frac{dy}{dt} = ?$

$\frac{dx}{dt} = 6\ \mathrm{ft}/s$

$t = 1$

$\Rightarrow x = 6$

$x^2 + (10 - y)^2 = 10^2$

$x^2 + 100 - 20y + y^2 = 100$

$x^2 - 20y + y^2 = 0$

$2x\frac{dx}{dt} - 20\frac{dy}{dt} + 2y\frac{dy}{dt} = 0$

$2(6)(6) + (2(2) - 20)\frac{dy}{dt} = 0$

$16\frac{dy}{dt} = 72$

$\frac{dy}{dt} = 4.5\ \mathrm{ft}/s$

$6^2 + (10 - y)^2 = 10^2$

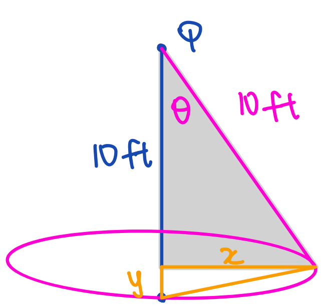
$36 + (10 - y)^2 = 100$

$(10 - y)^2 = 64$

$10 - y = 8$

$y = 2$

b. What is the angular speed of the rope in rad/s after 1 s?



$\frac{d\theta}{dt} = ?$

$\frac{dx}{dt} = 6\ \mathrm{ft}/s$

$t = 1$

$\Rightarrow x = 6$

$\sin\theta = \frac{x}{10}$

$\cos\theta\frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt} \times \frac{1}{\cos\theta}$

$= \frac{1}{\cancel{10}\,2}(\cancel{6}\,3)\left(\frac{\cancel{5}\,1}{\cancel{4}\,2}\right)$

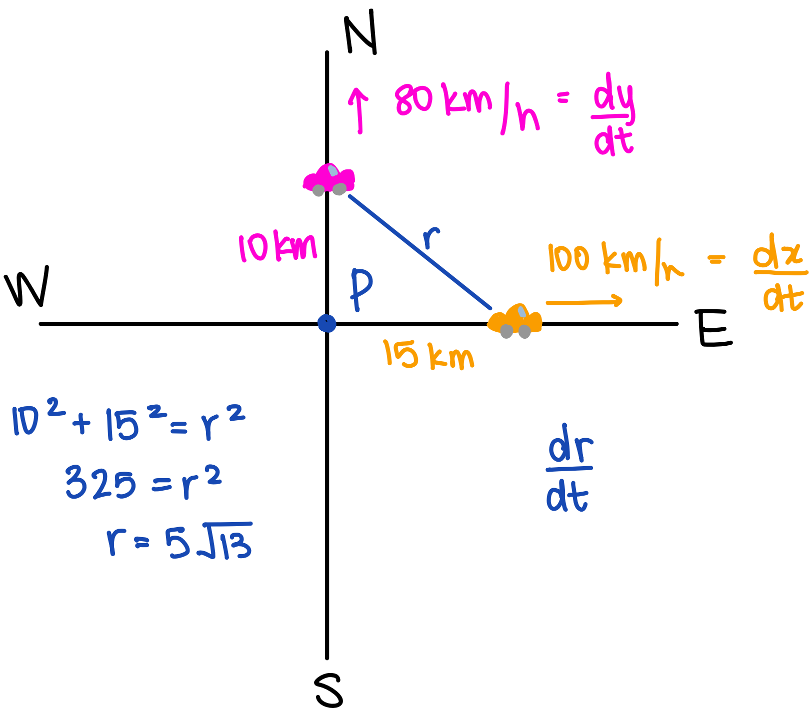
$= \frac{3}{4}$ RAD/s.

$\sin\theta = \frac{6}{10} = \frac{3}{5}$

$\cos\theta = \frac{8}{10} = \frac{4}{5}$

Assignment # 1 – 19 Quiz Next Day!

5. A road running north to south crosses a road going east to west at the point P. Car A is driving north along the first road and car B is driving east along the second road. At a particular time car A is 10 km to the north of P and travelling at $80\ \mathrm{km}/\mathrm{hr}$, while car B is $15\ km$ to the east of P and traveling at $100\ \mathrm{km}/\mathrm{hr}$. How fast is the distance between the two cars changing?



$$10^2 + 15^2 = r^2$$
$$325 = r^2$$
$$r = 5\sqrt{13}$$

$$x^2 + y^2 = r^2$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$$
$$2(15)(100) + 2(10)(80) = 2(5\sqrt{13})\frac{dr}{dt}$$
$$3000 + 1600 = 10\sqrt{13}\frac{dr}{dt}$$
$$\frac{460}{\sqrt{13}} = \frac{dr}{dt}$$
$$\frac{dr}{dt} = \frac{460\sqrt{13}}{13}\ \mathrm{km}/\mathrm{hr}$$