Name: _____ Date: _____

Learning Goal 3.2	Factoring, including the factor theorem and the remainder theorem.

Example Which of the following number is 7 a factor of? How do you know?

56 $= 7 \times 8$ R 0

92 $= 7 \times 13 + 1$

759 $= 7 \times 108 + 3$

812 $= 7 \times 116$

We can write them as a product w/ no remainder.

Example Find the remainder when $x^3 - 6x^2 + 7x + 6$ is divided by $x - 3$?

a. Synthetic/Long Division

b. Remainder Theorem $x = 3$

c. So is $x - 3$ a factor?

$$\begin{array}{c|cccc} 3 & 1 & -6 & 7 & 6 \\ & \downarrow & 3 & -9 & -6 \\ \hline & 1 & -3 & -2 & 0 \end{array}$$

↑ remainder.

$(3)^3 - 6(3)^2 + 7(3) + 6$
$= 27 - 54 + 21 + 6$
$= 0$

B/C the remainder is zero, $x-3$ is a factor.

$x^3 - 6x^2 + 7x + 6 = (x-3)(x^2 - 3x - 2) + 0$

The Factor Theorem	**Integral Zero Theorem**
A polynomial, in x, $P(x)$ has a factor of $x-a$ if $P(a) = 0$	if $x = a$ is an integral zero (integer solution) of a polynomial $P(x)$ with integral coefficients, the a has to be a factor of the constant term of $P(x)$
* can still use long/synthetic division	

Example Which of the following could be a factor of $x^3 + 5x^2 + 2x - 8$? $\pm 1, \pm 8, \pm 2, \pm 4$

$x + 2$ yes!

$x - 7$ no

$x - 8$ yes

$x + 16$ no (multiple vs. factor)

Show which of these, if any, is a factor.

$$\begin{array}{c|cccc} -2 & 1 & 5 & 2 & -8 \\ & \downarrow & -2 & -6 & 8 \\ \hline & 1 & 3 & -4 & 0 \end{array}$$

↑ Remainder = 0 $\Rightarrow$ $x+2$ is a factor

$x^3 + 5x^2 + 2x - 8 = (x+2)(x^2 + 3x - 4)$

$P(8) = (8)^3 + 5(8)^2 + 2(8) - 8$
$\neq 0$
so not a factor.

can't use synthetic division.

Example Verify that $2x - 3$ is a factor of $2x^3 - 5x^2 - x + 6$ in two different ways.

$$\begin{array}{r|l} & x^2 - x - 2 \\ \hline 2x-3 & 2x^3 - 5x^2 - x + 6 \\ & -(2x^3 - 3x^2) \\ \hline & -2x^2 - x \\ & -(-2x^2 + 3x) \\ \hline & -4x + 6 \\ & -(-4x + 6) \\ \hline & 0 \end{array}$$

$$P\left(\tfrac{3}{2}\right) = 2\left(\tfrac{3}{2}\right)^3 - 5\left(\tfrac{3}{2}\right)^2 - \tfrac{3}{2} + 6$$
$$= 2\left(\tfrac{27}{8}\right) - 5\left(\tfrac{9}{4}\right) - \tfrac{3}{2} + 6$$
$$= \tfrac{27}{4} - \tfrac{45}{4} - \tfrac{6}{4} + \tfrac{24}{4}$$
$$= 0$$

Because the remainder is zero, $2x-3$ is a factor

Example For what values of k will $x - 3$ be a factor of $2x^3 - kx^2 - 4x + 3$?

↳ $P(x)$

$$P(3) = 0 = 2(3)^3 - k(3)^2 - 4(3) + 3$$
$$0 = 54 - 9k - 12 + 3$$
$$0 = 45 - 9k$$
$$9k = 45$$
$$k = 5$$

Example Factor $x^3 - x^2 - 5x - 3$ fully.

By the integral zero theorem, the only possible integer factors will be $x+1$ and $x-1$ and $x+3$ and $x-3$

$$\begin{array}{r|cccc} -1 & 1 & -1 & -5 & -3 \\ & \downarrow & -1 & 2 & 3 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$$= (x+1)(x^2 - 2x - 3)$$

$-3 \times 1 = -3$, $-3 + 1 = -2$

$$= (x+1)(x-3)(x+1)$$
$$= (x+1)^2(x-3)$$