Name: ___________________________ Date: ____________________

Learning Goal 3.2	Factoring, including the factor theorem and the remainder theorem.

More Questions

1. For what values of a could $x-a$ be a factor of $f(x)=x^5+6x^4-5x^3-30x^2+4x+24$.

$$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$$

2. Eliminate any binomial that is not a factor of $f(x)$.

$$f(-1)=(-1)^5+6(-1)^4-5(-1)^3-30(-1)^2+4(-1)+24$$
$$=-1+6+5-30-4+24$$
$$=0$$

$x+1$

$$f(1)=(1)^5+6(1)^4-5(1)^3-30(1)^2+4(1)+24$$
$$=1+6-5-30+4+24$$
$$=0$$

$x-1$

Of a total possible 5 factors, we've found 2 – three left!

$$f(-2)=(-2)^5+6(-2)^4-5(-2)^3-30(-2)^2+4(-2)+24$$
$$=-32+6(16)-5(-8)-30(4)-8+24$$
$$=-32+96+40-120-8+24$$
$$=0$$

$x+2$

$$f(2)=(2)^5+6(2)^4-5(2)^3-30(2)^2+4(2)+24$$
$$=32+6(16)-5(8)-30(4)+8+24$$
$$=32+96-40-120+8+24$$
$$=0$$

$x-2$

Only one left! Also, if we're smart, we see the product of the constant terms so far is 4, so the last one is decided. If that doesn't occur to you, you'll do a little more algebra. It's cool ... just a waste of time 😉

$$f(-6)=(-6)^5+6(-6)^4-5(-6)^3-30(-6)^2+4(-6)+24$$
$$=-7776+6(1296)-5(-216)-30(36)-24+24$$
$$=-7776+7776+1080-1080-24+24$$
$$=0$$

$x+6$

of a total possible 5 factors, we've found 5 – we can stop!

3. Factor $f(x)$.

$$f(x)=x^5+6x^4-5x^3-30x^2+4x+24$$
$$=(x+1)(x-1)(x+2)(x-2)(x+6)$$

4. Factor $3y^3 + 13y^2 - 16$ fully.

The possible options for the constant term of the binomial are $\pm1, \pm2, \pm4, , \pm8, \pm16$ and there are 3 possibilities.

$y + 1$

$$3(-1)^3 + 13(-1)^2 - 16$$
$$= -3 + 13 - 16$$
$$= -6$$

$x - 1$

$$3(1)^3 + 13(1)^2 - 16$$
$$= 3 + 13 - 16$$
$$= 0$$

From here, you can keep checking factors, or divide out the factor you know to have a quadratic remaining. Just one more idea ... expecially since we know that one of them will **not** have a leading coefficient of 1.

1	3	13	0	−16
	↓	3	16	16
	3	16	16	0

$$3y^3 + 13y^2 - 16 = (x-1)(3x^2 + 16x + 16)$$

$12 \times 4 = 16 \times 3 = 48$

$12 + 4 = 16$

$$= (x-1)(3x^2 + 12x + 4x + 16)$$
$$= (x-1)[3x(x+4) + 4(x+4)]$$
$$= (x-1)(x+4)(3x+4)$$

5. Determine the value(s) of k so that the binomial is a factor of the polynomial.

a. $P(x) = x^3 + 5x^2 + kx + 6$

$x + 2$

$$P(-2) = (-2)^3 + 5(-2)^2 + k(-2) + 6 = 0$$

$$-8 + 5(4) - 2k + 6 = 0$$
$$-8 + 20 - 2k + 6 = 0$$
$$18 - 2k = 0$$
$$18 = 2k$$
$$9 = k$$

b. $P(x) = kx^3 - 10x^2 + 2x + 3$

$x - 3$

$$P(3) = k(3)^3 - 10(3)^2 + 2(3) + 3 = 0$$

$$27k - 10(9) + 6 + 3 = 0$$
$$27k - 90 + 6 + 3 = 0$$
$$27k - 81 = 0$$
$$27k = 81$$
$$k = 3$$

6. The product of four integers is $x^4 + 7x^3 + 7x^2 - 15x$, where x is one of the integers. What are the possible expressions for the other three integers?

$$x^4 + 7x^3 + 7x^2 - 15x = x(x^3 + 7x^2 + 7x - 15)$$

So the possible solutions or $x -$ intercepts are $\pm 1, \pm 3, \pm 5, \pm 15$ and there are 3 possibilities.

$x + 1$

$$(-1)^3 + 7(-1)^2 + 7(-1) - 15$$
$$= -1 + 7 - 7 - 15$$
$$= -16$$

the algebra is simple, but it doesn't eliminate any factors to use ± 1 first.

$x - 1$

$$(1)^3 + 7(1)^2 + 7(1) - 15$$
$$= 1 + 7 + 7 - 15$$
$$= 0$$

$x + 3$

$$(-3)^3 + 7(-3)^2 + 7(-3) - 15$$
$$= -27 + 7(9) - 21 - 15$$
$$= 0$$

So far the product is $3 \times -1 = -3$ so the other factor must be $x + 5$ ... but I'm still going to check!

$x + 5$

$$(-5)^3 + 7(-5)^2 + 7(-5) - 15$$
$$= -125 + 7(25) - 35 - 15$$
$$= 0$$

$$x^4 + 7x^3 + 7x^2 - 15x = x(x^3 + 7x^2 + 7x - 15)$$
$$= x(x - 1)(x + 3)(x + 5)$$

Assignment p. 133 # 1 – 7, 11 – 16 Quiz Next Day!