Name: ________________________ Date: __________________

Learning Goal 3.1	Using all basic derivative rules.

More Questions

1. Determine the 'inner' and 'outer' functions, then find the derivative.

a. $y = (5x^3 + 12x^2 - 15)^{-1}$

Outside

$f(x) = x^{-1}$

$f'(x) = -\frac{1}{x^2}$

Inside

$g(x) = 5x^3 + 12x^2 - 15$

$g'(x) = 15x^2 + 24x$

$$\frac{dy}{dx} = -\frac{1}{(5x^3 + 12x^2 - 15)^2} \times (15x^2 + 24x)$$

$$= -\frac{3x(5x+8)}{(5x^3 + 12x^2 - 15)^2}$$

b. $f(x) = \frac{1}{\sqrt{625 - x^2}}$

Outside

$g(x) = x^{-1/2}$

$g'(x) = -\frac{1}{2\sqrt{x^3}}$

$= -\frac{\sqrt{x}}{2x^2}$

Inside

$h(x) = 625 - x^2$

$h'(x) = -2x$

$$f'(x) = -\frac{\sqrt{625 - x^2}}{2(625 - x^2)^2} \times -2x$$

$$= \frac{x\sqrt{625 - x^2}}{(625 - x^2)^2}$$

c. $g(x) = \frac{x^2 - 1}{x\sqrt{x^2 + 1}}$

$$g'(x) = \frac{\left(x\sqrt{x^2+1}\right)(x^2-1)' - (x^2-1)\left(x\sqrt{x^2+1}\right)'}{\left(x\sqrt{x^2+1}\right)^2}$$

$(x^2 - 1)' = 2x$

$$\left(x\sqrt{x^2+1}\right)'$$

$$= \sqrt{x^2+1} + x\left(\frac{1}{2\sqrt{x^2+1}} \times 2x\right)$$

$$= \sqrt{x^2+1} + x^2\left(\frac{1}{\sqrt{x^2+1}}\right)$$

$$= \frac{2x^2+1}{\sqrt{x^2+1}}$$

$$g'(x) = \frac{\left(x\sqrt{x^2+1}\right)(2x) - (x^2-1)\frac{2x^2+1}{\sqrt{x^2+1}}}{\left(x\sqrt{x^2+1}\right)^2}$$

$$g'(x) = \frac{\left(x\sqrt{x^2+1}\right)(2x) - (2x^2+1)\sqrt{x^2+1}}{x^2(x^2+1)}$$

$$g'(x) = \frac{\sqrt{x^2+1}}{x^2(x^2+1)}$$

d. $y = \left(\frac{2x+1}{3x+2}\right)^3$

Outside

$f(x) = x^3$

$f'(x) = 3x^2$

Inside

$$g(x) = \frac{2x+1}{3x+2}$$

$$g'(x) = \frac{(3x+2)(2) - (2x+1)(3)}{(3x+2)^2}$$

$$= \frac{6x + 4 - 6x - 3}{(3x+2)^2}$$

$$= \frac{1}{(3x+2)^2}$$

$$y' = 3\left(\frac{2x+1}{3x+2}\right)^2 \times \frac{1}{(3x+2)^2}$$

$$= 3\frac{(2x+1)^2}{(3x+2)^4}$$

e. $y = \sqrt{1 + \sqrt{1 + \sqrt{x}}}$

Outside

$$f(x) = \sqrt{x}$$
$$f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2}$$

Inside

$$g(x) = 1 + \sqrt{x}$$
$$g'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2}$$

$$y = (f \circ g \circ g)(x)$$
$$y' = f'\big((g \circ g)(x)\big) \times g'\big(g(x)\big) \times g'(x)$$

$$= \frac{1}{2}\left(1 + \sqrt{1 + \sqrt{x}}\right)^{-1/2} \times \frac{1}{2}(1 + \sqrt{x})^{-1/2} \times \frac{1}{2}x^{-1/2}$$
$$= \frac{1}{8\sqrt{x \times \left(1 + \sqrt{x}\right) \times \left(1 + \sqrt{1 + \sqrt{x}}\right)}}$$

f. $h(x) = \sqrt{(x^2 + 1)^2 + \sqrt{1 + (x^2 + 1)^2}}$

Outside

$$f(x) = \sqrt{x}$$

Inside

$$g(x) = (x^2 + 1)^2 + \sqrt{1 + (x^2 + 1)^2}$$

Then there are two more inside and outside functions in $g(x)$::

Outside

$$a(x) = x^2$$

Inside

$$b(x) = x^2 + 1$$

Outside

$$m(x) = \sqrt{x}$$

Inside

$$n(x) = 1 + (x^2 + 1)^2$$

Be methodical … to get

$$h'(x) = \frac{1}{2\sqrt{g(x)}} \times \left(4x(x^2 + 1) + \frac{4x(x^2 + 1)}{2\sqrt{n(x)}}\right)$$

g. $y = \left(\frac{x - 2}{2x + 1}\right)^9$

Outside

$$f(x) = x^9$$
$$f'(x) = 9x^8$$

Inside

$$g(x) = \frac{x - 2}{2x + 1}$$
$$g'(x) = \frac{(2x + 1)(1) - (x - 2)(2)}{(2x + 1)^2} = \frac{2x + 1 - 2x + 4}{(2x + 1)^2} = \frac{5}{(2x + 1)^2}$$

$$\frac{dy}{dx} = 9\left(\frac{x - 2}{2x + 1}\right)^8 \times \frac{5}{(2x + 1)^2} = \frac{45(x - 2)^8}{(2x + 1)^{10}}$$

h. $h(x) = (2x + 1)(x^3 - x + 1)^4$

Perhaps try this one without identifying the inside and outside functions – it's a good one to start with!

$$h(x) = (2x + 1)'(x^3 - x + 1)^4 + (2x + 1)((x^3 - x + 1)^4)'$$
$$= 2(x^3 - x + 1)^4 + (2x + 1)(4(x^3 - x + 1)^3 \times (3x^2 - 1))$$
$$= 2(x^3 - x + 1)^3(13x^3 + 6x^2 - 5x - 1)$$

2. Find an equation for a tangent line at $(2,-7)$ to

$$\frac{x^2+x+1}{1-x}$$

$$\begin{aligned}\frac{d}{dx}\left(\frac{x^2+x+1}{1-x}\right) &= \frac{(1-x)(2x+1)-(x^2+x+1)(-1)}{(1-x)^2}\\ &= \frac{(-2x^2+x+1)+(x^2+x+1)}{(1-x)^2}\\ &= \frac{-x^2+2x+2}{(1-x)^2}\end{aligned}$$

At $x=2$, the slope of the tangent is

$$\frac{-(2)^2+2(2)+2}{\big(1-(2)\big)^2}=2$$

So the equation of the tangent at that point is

$$y+7=2(x-2)$$

3. Let $g(1)=2,\ f(2)=3,\ g'(1)=4$ and $f'(2)=5$. Find the derivative of $(f\circ g)(1)$.

$$\begin{aligned}(f\circ g)'(1) &= f'\big(g(1)\big)\times g'(1)\\ &= f'(2)\times g'(1)\\ &= 5\times 4\\ &= 20\end{aligned}$$