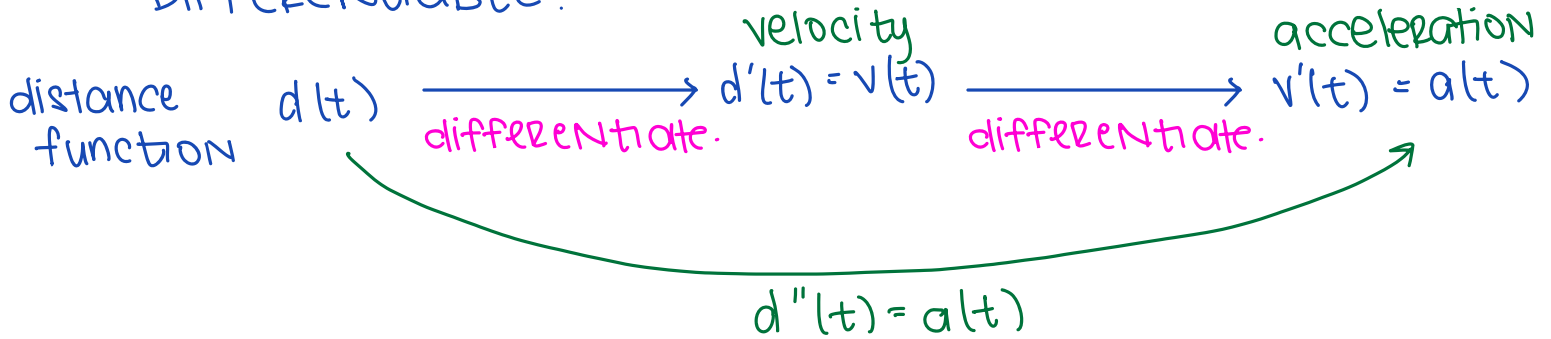


Name: _____

Date: _____

Learning Goal 3.3	Using more derivative rules.
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You can take the derivative of a derivative ... as many times as you want as long as what you are differentiating is differentiable!



Example Find $y^{(4)}$ of $y = x^3 - 6x^2 - 5x + 3$.

take the derivative 4 times.

$$\frac{dy}{dx} = 3x^2 - 12x - 5 \quad \text{derivative}$$

$$\frac{d^2y}{dx^2} = y'' = 6x - 12$$

$$\frac{d^3y}{dx^3} = y''' = 6$$

$$\frac{d^4y}{dx^4} = y^{(4)} = 0$$

Example Find $f^{(n)}(x)$ if $f(x) = \frac{1}{x} = x^{-1}$

any # of derivatives

⇒ find a pattern

$$f'(x) = -x^{-2} \quad -1 \quad (-1)!$$

$$f''(x) = -(-2)x^{-3} = 2x^{-3} \quad 2 = (-1)(-2) = (-2)!$$

$$f'''(x) = 2(-3)x^{-4} = -6x^{-4} \quad -6 = (-1)(-2)(-3) = (-3)!$$

in general $f^{(n)}(x) = (-1)^n n! x^{-(n+1)}$

$$f^{(4)}(x) = (-1)^4 (4)! x^{-(4+1)} = 24x^{-5}$$

Example Find y'' if $x^4 + y^4 = 16$.

FIRST $4x^3 + 4y^3 \frac{dy}{dx} = 0$

$$4y^3 \frac{dy}{dx} = -4x^3$$

$$\frac{dy}{dx} = \frac{-4x^3}{4y^3}$$

$$= \frac{-x^3}{y^3} = -\left(\frac{x}{y}\right)^3$$

Example Find $\frac{d^{27}}{dx^{27}}(\cos x)$.

$$\frac{d}{dx}(\cos x) = -\sin x = \frac{d^{25}}{dx^{25}}$$

$$\frac{d^2}{dx^2}(\cos x) = \frac{d}{dx}(-\sin x)$$

$$= -\cos x = \frac{d^{26}}{dx^{26}}$$

$$\frac{d^3}{dx^3}(\cos x) = \sin x = \frac{d^{27}}{dx^{27}}$$

$$\frac{d^4}{dx^4}(\cos x) = \cos x = \text{ORIGINAL} = \frac{d^8}{dx^8}$$

SECOND

$$\frac{d^2 y}{dx^2} = \frac{-3x^2 y^3 - 3y^2 x^3 \frac{dy}{dx}}{y^6}$$

$$= \frac{-3x^2 y^3 - 3y^2 x^3 \left(\frac{-x^3}{y^3}\right)}{y^6}$$

$$= \frac{-3x^2 y^3 + \frac{3x^6}{y}}{y^6}$$

$$= \frac{-3x^2 y^4 + 3x^6}{y^7}$$

$$\frac{d^{27}}{dx^{27}}(\cos x) = \sin x.$$