Name: ___________________________ Date: ____________________

Learning Goal 3.3	Using more derivative rules.

You can take the derivative of a derivative … as many times as you want as long as what you are differentiating is differentiable!

distance function $d(t) \xrightarrow{\text{differentiate}}$ velocity $d'(t) = v(t) \xrightarrow{\text{differentiate}}$ acceleration $v'(t) = a(t)$

$d''(t) = a(t)$

Example Find $y^{(4)}$ of $y = x^3 - 6x^2 - 5x + 3$.

take the derivative 4 times.

$$\frac{dy}{dx} = 3x^2 - 12x - 5$$

derivative

$$\frac{d^2y}{dx^2} = y'' = 6x - 12$$

$$\frac{d^3y}{dx^3} = y''' = 6$$

$$\frac{d^4y}{dx^4} = y^{(4)} = 0$$

Example Find $f^{(n)}(x)$ if $f(x) = \frac{1}{x}$. $= x^{-1}$

any # of derivatives

$\Rightarrow$ find a pattern

Factorial

$$f'(x) = -x^{-2} \qquad -1 \qquad (-1)!$$

$$f''(x) = -(-2)x^{-3} = 2x^{-3} \qquad 2 = (-1)(-2) = (-2)!$$

$$f'''(x) = 2(-3)x^{-4} = -6x^{-4} \qquad -6 = (-1)(-2)(-3) = (-3)!$$

in general $f^{(n)}(x) = (-1)^n n! x^{-(n+1)}$

$$f^{(4)}(x) = (-1)^4 (4)! x^{-(4+1)} = 24x^{-5}$$

Example Find y'' if $x^4 + y^4 = 16$.

FIRST

$$4x^3 + 4y^3\frac{dy}{dx} = 0$$

$$4y^3\frac{dy}{dx} = -4x^3$$

$$\frac{dy}{dx} = \frac{-4x^3}{4y^3} = \frac{-x^3}{y^3} = -\left(\frac{x}{y}\right)^3$$

SECOND

$$\frac{d^2y}{dx^2} = \frac{-3x^2y^3 - 3y^2x^3\frac{dy}{dx}}{y^6}$$

$$= \frac{-3x^2y^3 - 3\cancel{y^2}x^3\left(\frac{-x^3}{\cancel{y^3}}\right)}{y^6}$$

$$= \frac{-3x^2y^3 + \frac{3x^6}{y}}{y^6}$$

$$= \frac{-3x^2y^4 + 3x^6}{y^7}$$

Example Find $\frac{d^{27}}{dx^{27}}(\cos x)$.

$$\frac{d}{dx}(\cos x) = -\sin x = \frac{d^{25}}{dx^{25}}$$

$$\frac{d^2}{dx^2}(\cos x) = \frac{d}{dx}(-\sin x)$$

$$= -\cos x = \frac{d^{26}}{dx^{26}}$$

$$\frac{d^3}{dx^3}(\cos x) = \sin x = \frac{d^{27}}{dx^{27}}$$

$$\frac{d^4}{dx^4}(\cos x) = \cos x = \text{original} = \frac{d^8}{dx^8} = \frac{d^{12}}{dx^{12}} = \frac{d^{16}}{dx^{16}} = \frac{d^{20}}{dx^{20}} = \frac{d^{24}}{dx^{24}}$$

$$\frac{d^{27}}{dx^{27}}(\cos x) = \sin x.$$

Assignment # 1 – 37 Quiz Next Day!