Name: ____________________________ Date: ________________

Learning Goal 4.3	Evaluate expressions with fractional and negative exponents. Connect fractional exponents to radicals, and negative exponents to reciprocals.

Recall:

1. $3^6 3^2 =$

$3\times3\times3\times3\times3\times3\times3\times3$

$= 3^{6+2}$

$= 3^8$

2. $6^3 6^7 6^2 6^5 =$

$= 6^{3+7+2+5}$

$= 6^{17}$

3. $7^6 7^3 7^1 =$

$= 7^{6+3+1}$

$= 7^{10}$

Extend the idea to ~~non-whole~~ rational number exponents:

4. $2^{\frac{1}{2}} 2^{\frac{1}{2}} =$

$= 2^{\frac{1}{2}+\frac{1}{2}}$

$= 2^1$

$= 2$

5. $5^{0.25} 5^{0.25} 5^{0.25} 5^{0.25} =$

$= 5^{0.25+0.25+0.25+0.25}$

$= 5^1$

$= 5$

6. $11^{\frac{1}{3}} 11^{\frac{1}{3}} 11^{\frac{1}{3}} =$

$= 11^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$

$= 11^1$

$= 11$

Take a silent moment. What do you think the **fractional exponents** represent?

The denominator of your exponent is the index of your radical.

When *n* is a natural number and *x* is a rational number,

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

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1000

10 100

Examples

1. $1000^{\frac{1}{3}}$

$= (2^3 \times 5^3)^{\frac{1}{3}}$

$= (2^3)^{\frac{1}{3}}(5^3)^{\frac{1}{3}}$

$= (2^{\frac{3}{3}})(5^{\frac{3}{3}})$

$= 2 \times 5$

2. $0.25^{\frac{1}{2}}$

$= \left(\frac{25}{100}\right)^{\frac{1}{2}}$

$= \left(\frac{5^2}{2^2 \times 5^2}\right)^{\frac{1}{2}}$

$= \frac{5^1}{2^1 \times 5^1} = \frac{5}{10} = \frac{1}{2}$

3. $(-8)^{\frac{1}{3}} = -2$

$= ((-2)^3)^{\frac{1}{3}}$

$= (-2)^{3/3}$

$= -2$

4. $\left(\frac{16}{81}\right)^{\frac{1}{4}}$

$= \left(\frac{2^4}{3^4}\right)^{\frac{1}{4}}$

$= \frac{2^{\cancel{4/4}\,1}}{3^{\cancel{4/4}\,1}} = \frac{2}{3}$

What if the exponent is not a unit fraction? Take a silent minute to consider.

$(40^2)^{\frac{1}{3}} = 40^{\frac{2}{3}} =$

$40^{\frac{2}{3}} = \sqrt[3]{(40)^2}$	$40^{\frac{2}{3}} = (\sqrt[3]{40})^2$

When *m* and *n* are natural numbers, and *x* is a rational number,

$$x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

Examples

1. $0.01^{\frac{3}{2}}$

$= \left(\frac{1}{100}\right)^{\frac{3}{2}}$

$= \left(\sqrt{\frac{1}{100}}\right)^3 = \sqrt{\left(\frac{1}{100}\right)^3}$

$= \left(\frac{1}{10}\right)^3 = \sqrt{\frac{1}{1000\,000}}$

$= \frac{1^3}{10^3} = \frac{\sqrt{1}}{\sqrt{1000\,000}}$

$= \frac{1}{1000} = \frac{1}{1000}$

2. $(-27)^{\frac{4}{3}}$

$= (\sqrt[3]{-27})^4$

$= (-3)^4$

$= 81$

3. $81^{\frac{3}{4}}$

$= (\sqrt[4]{81})^3$

$= (3)^3$

$= 27$

prime factor tree.

$= \sqrt[4]{(81)^3}$

$= \sqrt[4]{(3^4)^3}$

$= \sqrt[4]{3^{12}}$

$= 3^{12/4}$

$= 3^3$

$= 27$

4. $0.75^{1.2}$

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$$0.25^{\frac{1}{2}} = \left(\frac{1}{4}\right)^{\frac{1}{2}} = \left(\frac{1}{4}\right)^{\frac{1}{2}} = \frac{1^{\frac{1}{2}}}{4^{\frac{1}{2}}} = \frac{1}{2}$$

$$\sqrt[2]{4}$$

$$= \left(\frac{1}{2^2}\right)^{\frac{1}{2}}$$

$$= \frac{1^{\frac{1}{2}}}{2^{\cancel{\frac{2}{2}}\,1}}$$

$$= \frac{1}{2}$$

$$0.75^{1.2} = \left(\frac{3}{4}\right)^{\frac{6}{5}} = \left(\frac{3}{4}\right)^{\cancel{\frac{5}{5}}\,1}\left(\frac{3}{4}\right)^{\frac{1}{5}} = \frac{3}{4}\sqrt[5]{\frac{3}{4}}$$

$$= \cancel{\frac{3\sqrt[5]{3}}{4\sqrt[5]{4}}}$$

$$= \left(\frac{75}{100}\right)^{\frac{6}{5}}$$