1. Vertex form.

$$y = ax^2 + bx + c$$

$$= a\left(x^2 + \frac{b}{a}x\right) + c \qquad \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c \times \frac{4a}{4a}$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a} = 0$$

$$+\frac{b^2 - 4ac}{4a} \qquad \frac{+b^2 - 4ac}{4a}$$

2. Solve

$$\frac{a}{a}\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a} \div a$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Given an equation in standard form, another method to solve is the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = ax^2 + bx + c$$

$b^2 - 4ac > 0$: 2 solutions

discriminant

< 0 : 0 solutions

$= 0$: 1 solution

Learning Goal 4.1	Given a quadratic equation, identify the number of solutions, zeros, roots or x – intercepts.

Example Use the discriminant to determine the nature of the roots for each quadratic equation, then solve.

a. $3x^2 + 4x + \frac{4}{3} = 0$

$b^2 - 4ac = (4)^2 - 4(3)\left(\frac{4}{3}\right)$

$= 16 - 16$

$= 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ (discriminant = 0)

$= \frac{-4}{2(3)}$

$= \frac{-4}{6}$

$= -\frac{2}{3}$

b. $2x^2 - 8x = -9$

$2x^2 - 8x + 9 = 0$

$b^2 - 4ac = (-8)^2 - 4(2)(9)$

$= 64 - 72$

$= -8$

$\Rightarrow$ no real solutions.

Learning Goal 4.2	Given a quadratic equation, find the values of solution(s) by factoring, completing the square or using the quadratic formula.

Example Determine the roots for each equation. Express exact answers.

a. $\frac{t^2}{4} - t - \frac{5}{2} = 0$ $\times 4$

$t^2 - 4t - 10 = 0$

$b^2 - 4ac = (-4)^2 - 4(1)(-10)$

$= 16 + 40$

$= 56 \Rightarrow$ 2 solutions

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-4) \pm \sqrt{56}}{2(1)} = \frac{4 \pm 2\sqrt{14}}{2}$

$= 2 \pm \sqrt{14}$

56 = 4 × 14

$\sqrt{56} = 2\sqrt{14}$

b. $3x^2 + 5x - 2 = 0$

$b^2 - 4ac = (5)^2 - 4(3)(-2)$

$= 25 + 24$

$= 49 \Rightarrow$ 2 solutions

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(5) \pm \sqrt{49}}{2(3)}$

$x_1 = \frac{-5+7}{6} = \frac{2}{6} = \frac{1}{3}$

$x_2 = \frac{-5-7}{6} = \frac{-12}{6} = -2$

Example Solve $6x^2 - 14x + 8 = 0$ by factoring, completing the square and by the quadratic formula.

① Factoring

$2(3x^2 - 7x + 4) = 0$

$2(3x^2 - 3x - 4x + 4) = 0$

$2[3x(x-1) - 4(x-1)] = 0$

$2(x-1)(3x-4) = 0$

$x - 1 = 0$ or $3x - 4 = 0$

$x = 1$ or $3x = 4$, $x = \frac{4}{3}$

② Completing the Square.

$6(x^2 - \frac{14}{6}x) + 8 = 0$

$(-\frac{14}{12})^2 = (-\frac{7}{6})^2 = \frac{49}{36}$

$6(x^2 - \frac{7}{3}x + \frac{49}{36} - \frac{49}{36}) + 8 = 0$

$6(x^2 - \frac{7}{3}x + \frac{49}{36}) - \frac{49}{6} + \frac{48}{6} = 0$

$6(x - \frac{7}{6})^2 - \frac{1}{6} = 0$

$6(x - \frac{7}{6})^2 = \frac{1}{6}$

$(x - \frac{7}{6})^2 = \frac{1}{36}$

$x - \frac{7}{6} = \pm \frac{1}{6}$

③ Quadratic Formula

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-14) \pm \sqrt{(-14)^2 - 4(6)(8)}}{2(6)}$

$= \frac{14 \pm \sqrt{196 - 192}}{12}$

$= \frac{14 \pm \sqrt{4}}{12}$

$= \frac{14 \pm 2}{12}$

$x = \frac{14+2}{12} = \frac{16}{12} = \frac{4}{3}$ or $x = \frac{14-2}{12} = \frac{12}{12} = 1$

$$x = \frac{7}{6} \pm \frac{1}{6}$$

$$x = \frac{7+1}{6} = \frac{8}{6} = \frac{4}{3}$$

$$x = \frac{7-1}{6} = \frac{6}{6} = 1$$