

Name: _____

Date: _____

Learning Goal 4.2

The Mean Value Theorem and L'Hospital's Rule

More Questions – Solutions

1. Evaluate.

$$\begin{aligned} \text{a. } \lim_{x \rightarrow \pi^-} \frac{\sin x}{1 - \cos x} & \\ \lim_{x \rightarrow \pi^-} \frac{\sin x}{1 - \cos x} &= \frac{\lim_{x \rightarrow \pi^-} \sin x}{\lim_{x \rightarrow \pi^-} 1 - \cos x} \\ &= \frac{0}{2} \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{b. } \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} & \\ \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} &= \frac{\lim_{x \rightarrow 3} (x^2 - 9)}{\lim_{x \rightarrow 3} (x - 3)} \\ &= \frac{0}{0} \\ &= \lim_{x \rightarrow 3} (2x) \\ &= \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} \\ &= \lim_{x \rightarrow 3} \frac{2x}{1} \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{c. } \lim_{x \rightarrow \infty} \frac{1 - x}{2x} & \\ \lim_{x \rightarrow \infty} \frac{1 - x}{2x} &= \frac{\lim_{x \rightarrow \infty} (1 - x)}{\lim_{x \rightarrow \infty} (2x)} \\ &= \frac{-\infty}{\infty} \\ &= \lim_{x \rightarrow \infty} \frac{-1}{2} \\ &= -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{d. } \lim_{x \rightarrow \infty} \frac{1 - x^2}{2x} & \\ \lim_{x \rightarrow \infty} \frac{1 - x^2}{2x} &= \frac{\lim_{x \rightarrow \infty} (1 - x^2)}{\lim_{x \rightarrow \infty} (2x)} \\ &= \frac{-\infty}{\infty} \\ &= \lim_{x \rightarrow \infty} \frac{-2x}{2} \\ &= -\infty \end{aligned}$$

$$e. \quad \lim_{x \rightarrow \pi} \frac{x^2 - \pi^2}{\sin x}$$

$$\begin{aligned} \lim_{x \rightarrow \pi} \frac{x^2 - \pi^2}{\sin x} &= \frac{\lim_{x \rightarrow \pi} (x^2 - \pi^2)}{\lim_{x \rightarrow \pi} (\sin x)} \\ &= \frac{0}{0} \\ &=^H \frac{\lim_{x \rightarrow \pi} (2x)}{\lim_{x \rightarrow \pi} (\cos x)} \\ &= \frac{2\pi}{-1} \\ &= -2\pi \end{aligned}$$

$$g. \quad \lim_{x \rightarrow 0} \frac{\sec x - 1}{\sin x}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sec x - 1}{\sin x} &= \frac{\lim_{x \rightarrow 0} (\sec x - 1)}{\lim_{x \rightarrow 0} (\sin x)} \\ &= \frac{0}{0} \\ &=^H \frac{\lim_{x \rightarrow 0} (\sec x \tan x)}{\lim_{x \rightarrow 0} (\cos x)} \\ &= \frac{0}{1} \\ &= 0 \end{aligned}$$

$$f. \quad \lim_{x \rightarrow \infty} \frac{2x^2 - 3x + 7}{x^2 + 47x + 1}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{2x^2 - 3x + 7}{x^2 + 47x + 1} &= \frac{\lim_{x \rightarrow \infty} (2x^2 - 3x + 7)}{\lim_{x \rightarrow \infty} (x^2 + 47x + 1)} \\ &= \frac{\infty}{\infty} \\ &=^H \frac{\lim_{x \rightarrow \infty} (4x - 3)}{\lim_{x \rightarrow \infty} (2x + 47)} \\ &= \frac{\infty}{\infty} \\ &=^H \frac{\lim_{x \rightarrow \infty} (4)}{\lim_{x \rightarrow \infty} (2)} \\ &= 2 \end{aligned}$$

$$h. \quad \lim_{x \rightarrow 0^+} \frac{1/x^2}{\ln x}$$

$$\begin{aligned} \lim_{x \rightarrow 0^+} \frac{1/x^2}{\ln x} &= \frac{\lim_{x \rightarrow 0^+} (1/x^2)}{\lim_{x \rightarrow 0^+} (\ln x)} \\ &= \frac{\infty}{-\infty} \\ &=^H \lim_{x \rightarrow 0^+} \frac{-2/x^3}{1/x} \\ &= \lim_{x \rightarrow 0^+} -\frac{2}{x^2} \\ &= -\infty \end{aligned}$$