Name: ________________________ Date: __________________

Learning Goal 5.1	Graphing primary trigonometric functions, including transformations and characteristics

More Questions - Solutions

1. At a seaport, the water has a maximum depth of 15 m at 7:00 a.m. The minimum depth of 5 m occurs 6.2 hours later. Assume the relation between the depth of the water and time is a sinusoidal function.

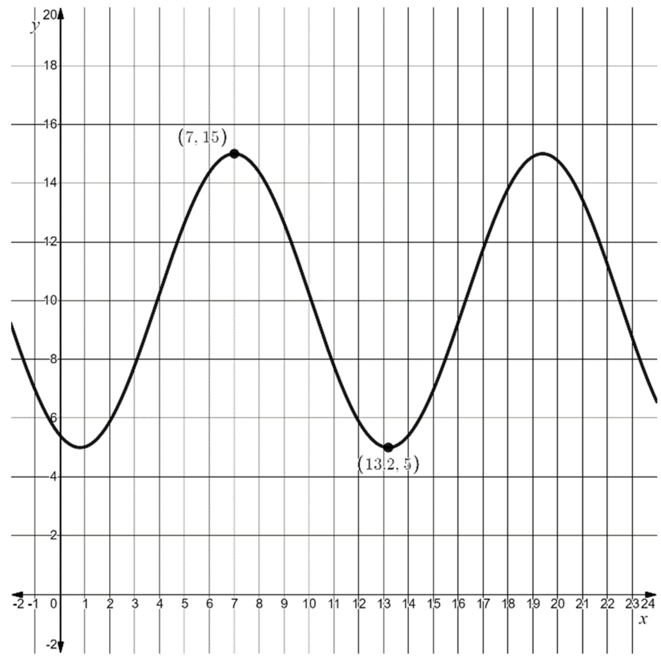
   a. Graph 24 hours of the tide cycle.

   b. Write an equation that expresses tide height as a function of the elapsed time, in the form
   $$h(t) = a\sin b(t-c)+d$$
   or
   $$h(t) = a\cos b(t-c)+d$$

$$a = \frac{15-5}{2} = \frac{10}{2} = 5$$

$$\frac{2\pi}{b} = 12.4 = \frac{62}{5}, \quad b = \frac{10\pi}{62} = \frac{5\pi}{31}$$

$$c = 5 + a = 5 + 5 = 10$$



$$y = 5\cos\frac{5\pi}{31}(x-7)+10$$

   c. What is the period of the function? 12.4 hours

   d. Estimate the depth at 11:00 a.m.

$$y = 5\cos\frac{5\pi}{31}(11-7)+10$$
$$y = 5\cos\frac{5\pi}{31}(4)+10$$
$$y = 5\cos\frac{20\pi}{31}+10$$
$$y = 5\cos\frac{20\pi}{31}+10$$
$$y \approx 5(-0.44039)+10$$
$$y \approx 7.8 \text{ metres}$$

e. Estimate one of the times when the water is 11 m deep.

$$11 = 5\cos\frac{5\pi}{31}(x-7) + 10$$
$$1 = 5\cos\frac{5\pi}{31}(x-7)$$
$$\frac{1}{5} = \cos\frac{5\pi}{31}(x-7)$$
$$\cos^{-1}\left(\frac{1}{5}\right) = \frac{5\pi}{31}(x-7)$$
$$x - 7 = \frac{31}{5\pi}\cos^{-1}\left(\frac{1}{5}\right)$$
$$x = \frac{31}{5\pi}\cos^{-1}\left(\frac{1}{5}\right) + 7$$
$$x \approx 9.70$$
$$x \approx 9{:}42 \text{ am}$$

2. The level of a certain hormone in the blood is cyclical over a period of 60 days. The maximum quantity is $600\ \mu\text{L}$ and the minimum is $280\ \mu\text{L}$. It can be modelled by the equation

$$q = a\cos bt + c.$$

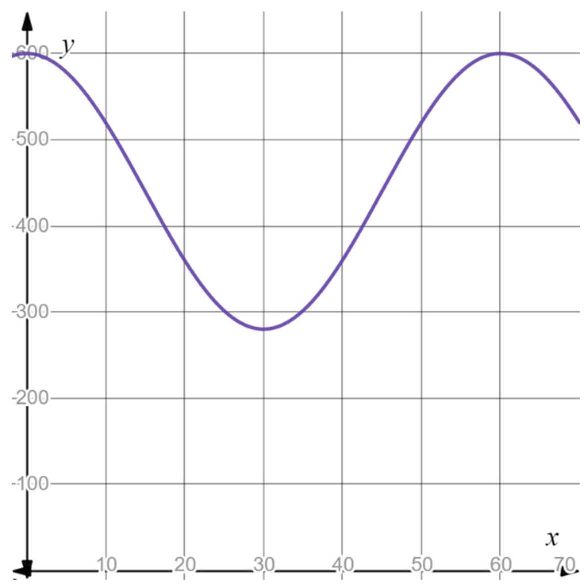
a. Sketch the graph to model this situation and hence find the values of a, b and c.

$$a = \frac{600-280}{2} = \frac{320}{2} = 160$$

$$\frac{2\pi}{b} = 60 \qquad b = \frac{2\pi}{60} = \frac{\pi}{30}$$

$$c = 280 + a = 280 + 160 = 440$$

$$q = 160\cos\frac{\pi}{30}t + 440$$



b. When the hormone level is below $350\ \mu\text{L}$ it can be a critical time for a genetically male person with a certain condition. For how long each cycle can a person with this condition be in a critical state?

$$350 = 160\cos\frac{\pi}{30}t + 440$$
$$-90 = 160\cos\frac{\pi}{30}t$$
$$-\frac{9}{16} = \cos\frac{\pi}{30}t$$
$$\cos^{-1}\left(-\frac{9}{16}\right) = \frac{\pi}{30}t$$
$$t = \frac{30}{\pi}\cos^{-1}\left(-\frac{9}{16}\right)$$
$$t = \frac{30}{\pi}\cos^{-1}\left(-\frac{9}{16}\right)$$

$$t_1 \approx \frac{30}{\pi}(2.168)$$
$$t_1 \approx \frac{30}{\pi}(2.168)$$
$$t_1 \approx 20.7$$

$$t_R \approx \pi - 2.168$$
$$t_R \approx 0.973$$

$$t_2 \approx \frac{30}{\pi}(\pi + t_R)$$
$$t_2 \approx \frac{30}{\pi}(\pi + 0.973)$$
$$t_2 \approx \frac{30}{\pi}(\pi + 0.973)$$
$$t_2 \approx \frac{30}{\pi}(4.115)$$
$$t_2 \approx 39.3$$

$$39.3 - 20.7 = 18.6$$

A person with this condition could be in a critical state for almost 19 days.